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Partial Sound Field Decomposition in Multi-Reference Nearfield Acoustical Holography by Using Optimally-Located Virtual References

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ABSTRACT

Multi-reference nearfield acoustical holography can be used to decompose the sound field radiated by a composite source into individual partial sound fields. To obtain physically meaningful partial fields, i.e., fields closely related to particular component sources, the reference microphones should be positioned as close as possible to the component sources. However, it is not always possible either to identify the optimal reference microphone locations prior to performing a holographic measurement, or to place reference microphones at those optimal locations, even if known. Here, post-processing procedures are described that make it possible both to identify the optimal reference locations and to place virtual references at those locations *after* performing a holographic measurement. The optimal reference microphone locations are defined to be those at which the MUSIC power is maximized in a three-dimensional space reconstructed by holographic projection. The signals at the locations thus identified can then be used as optimal "virtual" reference signals. It is shown through an experiment that the optimal virtual reference signals can be successfully used to identify physically meaningful partial sound fields.

1. INTRODUCTION

Nearfield acoustical holography (NAH) is a useful tool for visualizing noise sources and their associated sound fields since it allows sound fields that are measured on a surface to be projected through a three-dimensional space.[1] For the NAH procedure to be successful in its simplest form, the measured sound field must be spatially phase-coherent. Such a spatially phase-coherent sound field can be obtained by capturing the sound pressure on the entire measurement surface simultaneously. However, the latter measurement requires the use of many field microphones, perhaps hundreds or thousands, which may not always be practical.

A multi-reference procedure referred to as Spatial Transformation of Sound Fields (STSF), was introduced by Hald to accommodate situations in which the sound field is generated by a composite source comprising more than one mutually-incoherent source.[2] The latter procedure, described in terms of a reference cross-spectral matrix and a cross-spectral matrix linking the reference and field signals, is based on using singular value decomposition (SVD) to separate the total sound field into a set of phase-coherent partial fields that are incoherent with each other. The partial fields can then be independently projected and added together to yield the quadratic properties of the projected sound field. Note that in this procedure the characteristics of the partial fields depend on the reference locations with respect to the component sources, and thus the partial field decomposition is not unique. As a result, the partial fields determined in this way cannot usually be associated with particular component sources.

Since the introduction of STSF, a number of investigations have focused on separating the total sound field into individual partial sound fields that *can* be associated with meaningful physical sources. As an alternative to the SVD procedure, Hallman and Bolton[3] suggested the application of partial coherence decomposition (PCD) to separate the partial fields. Kwon and Bolton[4] compared the performance of the SVD and PCD methods; they also introduced the use of the multiple signal classification (MUSIC) algorithm[5] for the selection of the best references from amongst many candidates positioned around a sound source. It was shown that the reference microphones should be positioned close to the individual physical sources if physically meaningful partial sound fields were to be obtained. However, in practice, the location of the most prominent sources is sometimes not known prior to performing a holographic measurement. There are also cases in which it is impossible to place references at the desired locations even when those locations are known: for example, when a reference microphone placed close to a physical source might itself induce flow noise. Thus, to facilitate the decomposition of a sound field into physically meaningful components, it would be desirable to be able to place "virtual" references at optimal locations *after* performing a holographic measurement made using a set of sufficient, but non-optimally located references.

A virtual reference procedure was first described by Nam and Kim:[6] it was based on a vector representation[7] of the sound field combined with a knowledge of various point-to-point transfer functions both measured and estimated by using holographic projections. Nam and Kim also concluded that virtual references should be located at positions on the source surface where the amplitude of an acoustical property of interest, e.g., the pressure, particle velocity, or active intensity, was a maximum.[6] Through numerical simulations it was shown that by positioning the virtual references appropriately on the source surface the sound field could be decomposed into physically meaningful partial fields, particularly when PCD was performed.

In this article, the partial field decomposition procedure based on the use of virtual references introduced by Nam and Kim is re-derived in an alternative form, based on the use of signal cross-spectral matrices and transfer functions, with the intent of developing a procedure that is compatible with conventional multi-reference NAH techniques. The procedure to be described here, unlike that of Nam and Kim, also makes it possible to use a larger number of real than virtual references as would usually be the case in practice. An optimization procedure combined with the MUSIC algorithm was used to identify the optimal reference locations. The optimal reference locations were identified by searching for the positions in three-dimensional space at which the MUSIC power was locally maximized. It is shown here through an experiment that the optimal virtual reference procedure can yield improved estimates of the physical partial sound fields.

2. THEORY

A. Multi-reference NAH

When the total sound field generated by a composite source comprising a finite number of uncorrelated physical sources is completely sensed by a set of reference transducers (i.e., the number of references, M, is equal to or larger than the number of uncorrelated sources, K, ($K \le M$) and each uncorrelated source is sensed by at least one reference transducer), the field signals at the N measurement points located on the hologram surface can be expressed as a linear combination of reference signals multiplied by a N by M transfer function matrix, $\mathbf{H_{yr}}$.[4,9] Note that the number of incoherent sources in any particular case can be determined by counting the number of significant singular values when the reference cross-spectral matrix is decomposed by using SVD.[8] When the singular values associated with the noise subspace are small enough to be ignored, the reference cross-spectral matrix, $\mathbf{S_{rr}}$, can then be represented as the product of a K by K diagonal matrix, Λ, whose elements are the significant singular values, and a M by K unitary matrix, $\mathbf{V}$, whose column vectors are the eigenvectors of $\mathbf{S_{rr}}(\mathbf{S_{rr}})^{H}$: i.e.,

$$\mathbf{S_{rr}} = \mathbf{V\Lambda V}^{H}. \tag{1}$$

By using Eq. (1), the transfer matrix relating the reference and field signals can then be expressed as[9]

$$\mathbf{H_{yr}} = \mathbf{S_{ry}^{H} S_{rr}^{-1}} = \mathbf{S_{ry}^{H} V \Lambda^{-1} V}^{H}. \tag{2}$$

The cross-spectral matrix of the field signals on the hologram surface can be estimated by using both the reference cross-spectral matrix, $\mathbf{S_{rr}}$, in Eq. (1) and the transfer matrix, $\mathbf{H_{yr}}$, in Eq. (2): i.e.,

$$\mathbf{S_{yy}} = \mathbf{H_{yr} S_{rr} H_{yr}^{H}} = \mathbf{H_{yr} V \Lambda V}^{H} \mathbf{H_{yr}^{H}}. \tag{3}$$

The cross-spectral matrix of field signals on the hologram surface can then be decomposed to represent a set of incoherent partial fields subject only to the condition that $\mathbf{S_{yy}} = \mathbf{YY}^{H}$, where each column of the matrix, $\mathbf{Y}$, represents a partial field vector: i.e.,

$$\mathbf{Y} = \mathbf{H_{yr} V \Lambda}^{1/2} = \mathbf{H_{yl} \Lambda}^{1/2}, \tag{4}$$

where $\Lambda^{1/2}$ is the principal reference signal matrix and $\mathbf{H_{yl}}$ is the transfer matrix between the principal reference signal matrix and the partial field matrix. The sound field on any reconstruction surface can then be expressed as,

$$\mathbf{Y'} = \mathbf{H_{y'y} Y} = \mathbf{H_{y'y} H_{yr} V \Lambda}^{1/2} = \mathbf{H_{y'y} H_{yl} \Lambda}^{1/2} = \mathbf{H_{y'l} \Lambda}^{1/2}, \tag{5}$$

where $\mathbf{Y'}$ is the partial field matrix on the reconstruction surface and $\mathbf{H_{y'l}}$ represents the transfer matrix between the principal reference signals and the partial field signals on the reconstruction surface. The matrix $\mathbf{H_{y'y}}$ is composed of the transfer functions that relate the field signals on the hologram and reconstruction surfaces, and it represents the NAH projection procedure including the spatial Fourier transform (in the planar case) and the propagation operation. Note that each column of $\mathbf{H_{y'l}}$ corresponds to the NAH projection of the corresponding column of $\mathbf{H_{yl}}$. The cross-spectral matrix of the field signals on a reconstruction surface can then be calculated from Eq. (5) as

$$\mathbf{S_{y'y'}} = \mathbf{Y'Y'}^{H} = \mathbf{H_{y'l} \Lambda H_{y'l}^{H}}. \tag{6}$$

B. Virtual reference procedure

Virtual references can, in principle, be placed anywhere within the three-dimensional space covered by the NAH projection. By making use of the partial field signal matrices evaluated on the reconstruction surfaces, as in Eq. (5), the virtual reference signal matrix can be expressed as

$$\mathbf{X} = \begin{bmatrix} \mathbf{c}_1^{\mathrm{T}}\mathbf{Y}'_1 \\ \mathbf{c}_2^{\mathrm{T}}\mathbf{Y}'_2 \\ \vdots \\ \mathbf{c}_K^{\mathrm{T}}\mathbf{Y}'_K \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1^{\mathrm{T}}\mathbf{H}_{\mathbf{y}'_1\mathbf{l}} \\ \mathbf{c}_2^{\mathrm{T}}\mathbf{H}_{\mathbf{y}'_2\mathbf{l}} \\ \vdots \\ \mathbf{c}_K^{\mathrm{T}}\mathbf{H}_{\mathbf{y}'_K\mathbf{l}} \end{bmatrix}\mathbf{\Lambda}^{1/2} = \mathbf{H}_{\mathbf{xl}}\mathbf{\Lambda}^{1/2} \tag{7}$$

where $\mathbf{X}$ is the K by K virtual reference signal matrix. In Eq. (7), $\mathbf{c}_m$ represents the N by 1 reference selection vector: when the m-th virtual reference is positioned at the i-th field position on the reconstruction surface, all elements of $\mathbf{c}_m$ are zeros except for the element at the i-th row, which is itself unity. Note that the matrix $\mathbf{Y}'_m$ in Eq. (7) represents the partial field signal matrix for the reconstruction surface on which the m-th virtual reference is placed, and that the vector, $\mathbf{c}_m$, denotes the m-th virtual reference location on the m-th reconstruction surface. The cross-spectral matrices between the virtual reference signals, and between the virtual reference and the field signals on the hologram surface, can then be obtained from Eqs. (4) and (7): i.e.,

$$\mathbf{S}_{\mathbf{xx}} = \mathbf{X}\mathbf{X}^{\mathrm{H}} = \mathbf{H}_{\mathbf{xl}}\mathbf{\Lambda}\mathbf{H}_{\mathbf{xl}}^{\mathrm{H}}, \tag{8}$$

and

$$\mathbf{S}_{\mathbf{xy}} = \mathbf{X}\mathbf{Y}^{\mathrm{H}} = \mathbf{H}_{\mathbf{xl}}\mathbf{\Lambda}\mathbf{H}_{\mathbf{yl}}^{\mathrm{H}}. \tag{9}$$

Since the MUSIC algorithm can be used to find the locations of sources in a three-dimensional space,[5] the latter procedure has here been modified to identify the optimal virtual reference locations: the optimal locations are identified as the points in a three-dimensional space at which the MUSIC power, defined below, is maximized.

When the cross-spectral matrix of the field signals on a reconstruction surface, i.e., $\mathbf{S}_{\mathbf{y}'\mathbf{y}'}$ in Eq. (6), is decomposed by using the SVD procedure ($\mathbf{S}_{\mathbf{y}'\mathbf{y}'} = \mathbf{W}\Sigma\mathbf{W}^{\mathrm{H}}$), the unitary matrix, $\mathbf{W}$, can be expressed in terms of the eigenvectors of $\mathbf{S}_{\mathbf{y}'\mathbf{y}'}(\mathbf{S}_{\mathbf{y}'\mathbf{y}'})^{\mathrm{H}}$: i.e., $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2 \cdots \mathbf{w}_N]$, where $\mathbf{w}_n$ is the n-th eigenvector associated with the n-th singular value. Since the number of incoherent sources is K, the noise subspace, $\mathbf{R}_{\text{noise}}$, can be defined in terms of the noise-related eigenvectors, $\mathbf{w}_n$ ($n = K$+1 to N), as

$$\mathbf{R}_{\text{noise}} = \sum_{n=K+1}^{N}\mathbf{w}_n\mathbf{w}_n^{\mathrm{H}}. \tag{10}$$

The MUSIC power is then defined in terms of $\mathbf{R}_{\text{noise}}$ as

$$P_{\text{MUSIC}} = \frac{1}{\mathbf{u}^{\mathrm{H}}\mathbf{R}_{\text{noise}}\mathbf{u}}, \tag{11}$$

where $\mathbf{u}$ is the trial vector. Since the signal subspace spanned by $\mathbf{w}_n$ (n = 1 to K) is orthogonal to the noise subspace represented by $\mathbf{R}_{\text{noise}}$, the MUSIC power should be infinite when $\mathbf{u} = \mathbf{w}_n$ (n = 1 to K). Assume, for example, that a vector representing a source at the n-th discrete point of the reconstruction surface can be approximated as the trial vector,

$$\mathbf{u}_n = [0 \ \cdots \ 0 \ 1 \ 0 \ \cdots \ 0]^{\mathrm{T}}, \tag{12}$$

where the n-th element of $\mathbf{u}_n$ is one and the other N-1 elements are zeros. In that case, the MUSIC power associated with the trial vector, $\mathbf{u}_n$, would be very large. The trial vector defined in Eq. (12) closely represents the sound field generated by a monopole source. It is thus likely that the maximum MUSIC power procedure as described here will identify the locations of monopole-like sources.

After calculating the MUSIC powers associated with all possible trial vectors, $\mathbf{u} = \mathbf{u}_n$ (n = 1 to N), a map of the MUSIC power on a two-dimensional reconstruction surface can be obtained by reshaping the MUSIC power vector into a matrix whose elements represent the MUSIC power at a point on a two-dimensional surface. A three-dimensional MUSIC power image can then be obtained by repeating the latter operations on other projection surfaces in sequence. The optimal virtual reference locations are those where the MUSIC power is maximized locally. The virtual reference signals at those optimal locations can then be obtained from Eq. (7) since the reference locations that are identified by performing the optimal search are expressed in terms of the reconstruction surface locations and the reference selection vectors that appear in the latter equation.

C. Partial sound field decomposition

The cross-spectral matrices between the virtual reference signals, and between the virtual reference and field signals on the hologram surface, can be calculated from Eqs. (8) and (9). By using the SVD procedure, the virtual reference cross-spectral matrix is represented as $\mathbf{S_{xx}} = \mathbf{USU}^{\mathrm{H}}$, where $\mathbf{U}$ is the unitary matrix and $\mathbf{S}$ is the diagonal matrix of singular values. The partial fields can thus be represented as

$$\mathbf{Y}_{\mathbf{x},\,\mathrm{SVD}} = \mathbf{S}_{\mathbf{xy}}^{\mathrm{H}} \mathbf{U} \mathbf{S}^{-1/2} . \tag{13}$$

The PCD procedure is based on the use of LU decomposition to separate the reference cross-spectral matrix into two matrices: i.e., $\mathbf{S_{xx}} = \mathbf{LDL}^{\mathrm{H}}$, where $\mathbf{L}$ is the lower triangular matrix whose diagonal elements are unity and $\mathbf{D}$ is the diagonal matrix. In the latter case the partial fields are written as

$$\mathbf{Y}_{\mathbf{x},\,\mathrm{PCD}} = \mathbf{S}_{\mathbf{xy}}^{\mathrm{H}} \mathbf{L}^{\mathrm{H}^{-1}} \mathbf{D}^{-1/2} . \tag{14}$$

3. EXPERIMENT

A planar NAH experiment was performed in an anechoic chamber to illustrate the optimal virtual reference selection procedure described above. Two loudspeakers having different frequency characteristics were driven by independent white noise sources (see Fig. 1). A horizontal line array of nine microphones spaced 0.051 m apart was used to scan the hologram following the procedure indicated in Fig. 1. The aperture thus comprised 32 sub-holograms and a total of 18 by 16 field points. The source surface (i.e., the z = 0 m plane) coincided with the front surfaces of the loudspeakers. The field measurements were made on a hologram surface located at z = 0.05 m. During the scanning, five reference microphones were fixed in front of the loudspeakers as shown in Fig. 1. In each scan, the data record length was 512 points at a sampling rate of 4096 Hz and 20 linear averages were performed when estimating the various spectra; during the latter operations a 256 point overlap was used and a Hanning window was applied to each record.

4. RESULTS

The MUSIC power on the source plane (i.e., z = 0 m) is shown in Fig. 2(a) from 0 to 1600 Hz: a measurement point on the x-y plane is here represented by the vector index, n. In Fig. 2(b), the MUSIC power vector at 1200 Hz is plotted on the z = 0 plane: the vector was reshaped into a matrix whose i, j-th element represents the i, j-th measurement point on the x-y plane and the frequency was chosen arbitrarily. The two loudspeaker locations can easily be identified from the positions of the local maxima of the MUSIC power: those locations correspond to vector indices, n = 98 and n = 170, the positions of loudspeakers 1 and 2, respectively. The MUSIC power at 1200 Hz was then calculated on projection planes from z = -0.1 to 0.1 m as shown in Fig. 2(c). Local maxima of the MUSIC power were found at z = -0.021 m for n = 98 and at z = -0.005 m for n = 170.

The measured data at 1200 Hz was used for the purpose of comparing the characteristics of the partial fields decomposed by using either the SVD or PCD procedures combined with either the set of real or optimal virtual references. The decomposed partial pressure fields on the hologram surface (i.e., at z = 0.05 m) obtained by using the PCD procedure are shown in Fig. 3, when the measurement was made by using references 3 and 5. When real references 3 and 5 were used as the basis for the partial field decomposition, the decomposed partial fields do not represent the physical partial fields associated with the loudspeakers as may be seen in Figs. 3(a) and 3(b): i.e., both partial fields contain contributions from both loudspeakers. However, the partial fields decomposed on the basis of the optimal virtual references, are closely associated with the physical sources: see Figs. 3(c) and 3(d). Note however, that even in the latter cases there was a small amount of "leakage" from one partial field to another: i.e., a faint trace of the second source is visible in the first partial field (see Fig. 3(c)).

In order to compare the decomposed partial fields quantitatively, the amplitudes of the individual partial sound pressure fields at x = 0.356 m are plotted as a function of y-position in Fig. 4. In the first partial field, it can be seen that there is a contribution from the second loudspeaker at y = 0.457 m: the leakage was smaller when the sound fields were decomposed by using the PCD procedure rather than the SVD procedure, and when using the optimal virtual references rather than the real references 1 and 2 (see Fig. 4(a)). The pressure amplitudes at y = 0.457 m in the second partial field appear in the reverse order of the leakage amplitude in the first partial field (see Fig. 4(b)). The decomposed partial fields obtained by using virtual references placed either on the source surface or at the maximum MUSIC power locations are plotted in Fig. 5: the optimal virtual references placed at the maximum MUSIC power locations resulted in smaller leakage in the first field and a larger pressure amplitude at y = 0.457 m in the second partial field.

5. CONCLUSIONS

In this article, a post-processing procedure has been described that makes it possible to identify virtual reference signals that can be used to create physically meaningful partial fields *after* performing a holographic measurement based on a sufficient, but, non-optimally located reference set. The optimal virtual references were placed at the positions where the MUSIC power was locally maximized. It was shown through an experiment that the optimal virtual reference procedure results in partial fields that are very similar to the "real" partial fields associated with individual physical sources regardless of the locations of the real references. It was also found that using the set of virtual references located at the maximum MUSIC power locations resulted in more accurate estimates of the partial fields than were obtained using any other set of virtual references (e.g., on the source plane) or even a set of "good" real references (i.e., references 1 and 2). It was also shown that the partial fields obtained using the PCD procedure suffered less leakage than those obtained using the SVD procedure. Thus, it was found that the most meaningful partial fields were obtained when the PCD procedure was used in combination with optimally-located virtual references.

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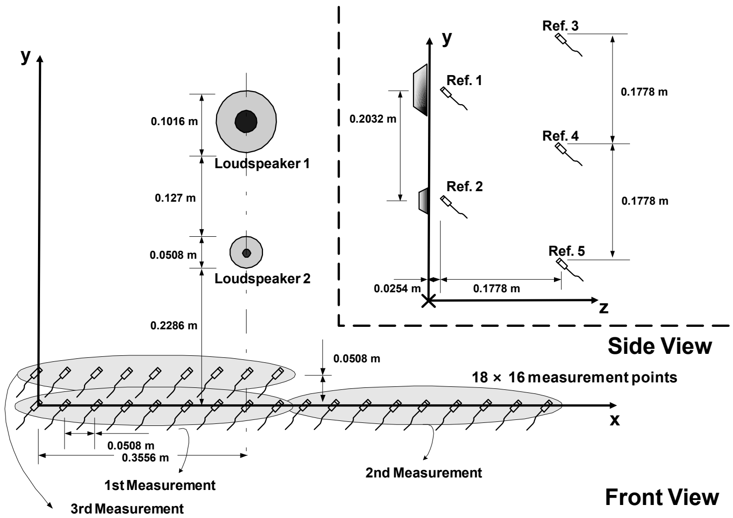

Figure 1. Sketch of experimental setup.

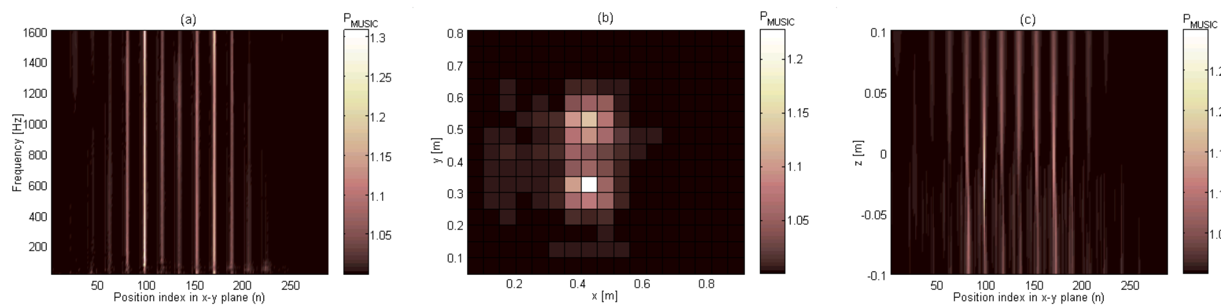

Figure 2. Music power on the source plane (z = 0 m) (experiment): (a) from 0 to 1600 Hz (maxima at n = 98 and 170), (b) mapped to the x-y plane at 1200 Hz, and (c) as a function of n = 1 to 288 and z = -0.1 to 0.1 m (when n = 98, MAX(P_{MUSIC}) = 1.298 at z = -0.021 m and when n = 170, MAX(P_{MUSIC}) = 1.135 at z = -0.005 m).

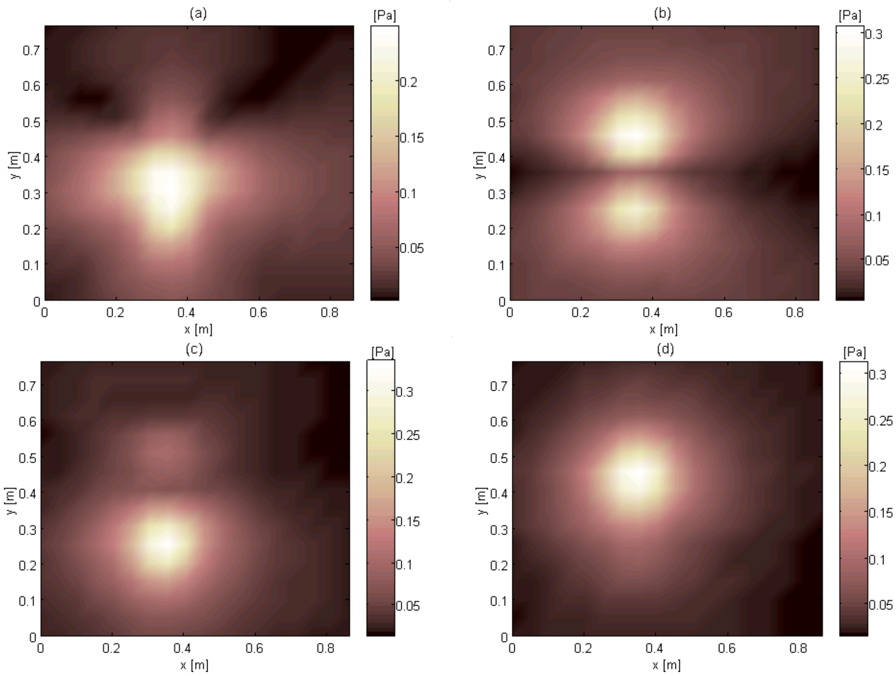

Figure 3. Amplitudes of decomposed partial pressure fields on the hologram surface at 1200 Hz when the measurement was made using references 3 and 5 (experiment): (a) 1st field (real references, PCD), and (b) 2nd field (real references, PCD), (c) 1st field (optimal virtual references, PCD), and (d) 2nd field (optimal virtual references, PCD).

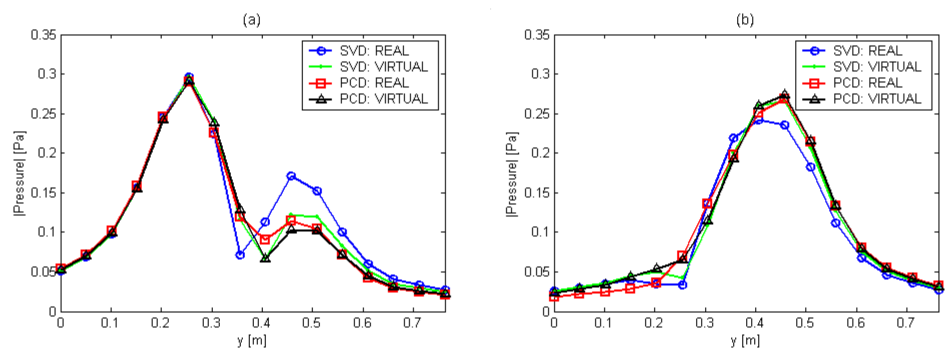

Figure 4. Amplitudes of partial sound pressure fields at x = 0.356 m as a function of y obtained by using sets of real references 1 and 2 and optimal virtual references: (a) first partial pressure field, and (b) second partial pressure field.

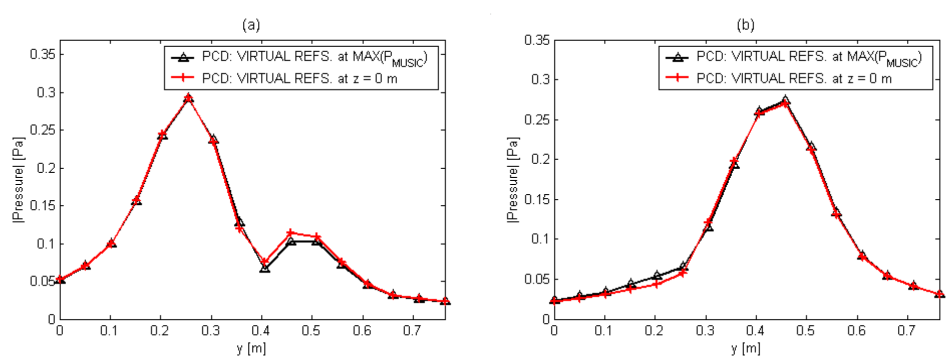

Figure 5. Amplitudes of partial sound pressure fields at x = 0.356 m as a function of y obtained by using virtual references placed on source surface and at the maximum MUSIC power locations: (a) first partial pressure field, and (b) second partial pressure field.