INTER-NOISE 2006
3-6 DECEMBER 2006
HONOLULU, HAWAII, USA

Identification of sound transmission characteristics of honeycomb sandwich panels using hybrid analytical/one-dimensional finite element method

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ABSTRACT

For the purpose of identifying the sound transmission characteristics of honeycomb sandwich panels that are commonly used in aircraft industries, Finite Element Method (FEM) combined with Boundary Element Method (BEM) has been widely used. However, the latter approach is not always applicable to high frequency analysis since it requires a large number of FEM/BEM meshes resulting in high computational cost. Although various analytical methods can also be used, their applications are restricted to few panels, e.g., that have simple layer configuration. Here, a hybrid analytical/one-dimensional FEM is described that uses finite element approximation in the thickness direction while analytical solutions are assumed in the plane directions. Thus, it makes possible to use small number of finite elements even for high frequency analysis in computationally efficient manner. The proposed method can be used to analyze the effects of boundary conditions at the edges of a panel. It can also be used to analyze various multi-layered panels of which each layer is represented by orthotropic material properties. As an application, the sound transmission characteristics of infinite-size panels are analyzed. By comparison with experimental data, it is shown that the proposed method successfully identify the sound transmission characteristics of honeycomb sandwich panels.

1 INTRODUCTION

Multi-layered, composite panels have been widely used in aircraft industries due to their superior mechanical properties (i.e., light weight and high strength). However, it is well known that their sound transmission characteristics are generally poor, which makes them unfavorable to interior noise. As interior noise has been increasingly emphasized, it has been much more desirable to investigate the sound transmission characteristics of multi-layered composite panels in detail.

For the purpose of investigating various composite panels in general, it is required to consider thick panels of which thickness is comparable to structural wave length at the maximum frequency of interest. It is also desired to consider various waves (e.g., flexural, shear, and longitudinal waves) propagating though the panels. In addition, it is required to account for orthotropic material properties: e.g., fiber materials are inserted to reinforce composite materials in particular directions.

To accommodate aforementioned aspects, various numerical and analytical methods have been developed. In particular, Finite Element Method (FEM) combined with boundary element method (BEM) has been widely used. However, the latter approach is not always applicable to relatively high frequency analysis since it is required to use a large number of FEM/BEM

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meshes resulting in high computational cost. Although various other methods can be used to analyze the sound transmission characteristics of multi-layered, composite panels in high frequency region, their applications are restricted to few composite panels that have simple layer configuration, such as sandwich panels, thin panels, or panels with isotropic or transverse-isotropic layers.

Here, a hybrid analytical/one-dimensional finite element method is described that uses finite element approximation in the thickness direction while analytical solutions are assumed in the plane directions. Thus, it makes possible to use small number of finite elements even for high frequency analysis in computationally efficient manner. Since the hybrid FE formulation presented in this article is expressed as spatial differential equation in the plane directions, it can be also used to analyze the effects of various boundary conditions, at the edges of a panel, which significantly affects sound transmission characteristics in low-frequency region. However, in this article, it is focused on infinite-sized panels so that sound transmission characteristics can be only dependent on the properties of panels (not on boundary conditions).

The hybrid analytical/one-dimensional finite element method is here applied to analyze the sound transmission characteristics of various honeycomb sandwich panels. Each layer of the panels is represented by a set of orthotropic material properties and total panel thickness can be large compared to the minimum structural wave length of interest. It is observed that the hybrid FE Transmission Loss (TL) predictions agree well with the measured data. In addition, TL sensitivity with respect to each design variable is presented that indicate how much each variable contributes to the TL results. As an application, the TL sensitivity results are here used to modify material properties to make the hybrid FE predictions match better with the experimental data. In order to understand the sound transmission characteristics in detail, both flexural and core shear waves propagating through the panels have been identified in the wave-number/frequency domain. It is found that in low frequency region, flexural wave dominates sound transmission behaviors for the honeycomb sandwich panels considered in this article: while in high frequency region, core shear wave mainly affects sound transmission characteristics. It is also shown that the critical frequency is generally located where the flexural wave is in transition to the core shear wave for the honeycomb panels.

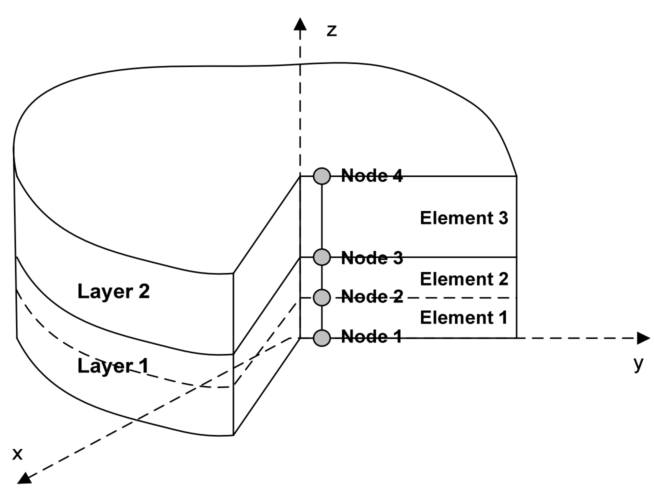

Figure 1: Illustration of hybrid FE model of double-layered panel.

2 FORMULATION OF HYBRID FINITE ELEMENT

For FE formulation, it is assumed that each layer is homogeneous: i.e., each layer is assumed to be represented by a single set of material properties. It is also assumed that each layer has constant thickness. Figure 1 illustrates a hybrid FE model of a double-layered panel. In this model, two elements are used to represent layer 1 and one element for layer 2. Since the displacements are approximated by the combination of nodal displacements and linear interpolation functions in the z-direction, there are two nodes per one element. In the following section, a hybrid analytical/one-dimension FE formulation is derived for a single element. The global equation of motion for a multi-element system can then be obtained by assembling the local element equation.

2.1 Equation of Motion

The displacements of a single hybrid element can be approximated in terms of the nodal displacements (which are the functions of *x*, *y*, and *t*) combined with linear interpolation functions in the z-direction [1]. Then, the equation of motion for the element can be derived using the procedure described in Ref. [2]: i.e.,

$$\mathbf{K}_{xx}\frac{\partial^2\mathbf{u}}{\partial x^2}+\mathbf{K}_{xy}\frac{\partial^2\mathbf{u}}{\partial x\partial y}+\mathbf{K}_{yy}\frac{\partial^2\mathbf{u}}{\partial y^2}+\mathbf{K}_{xz}\frac{\partial\mathbf{u}}{\partial x}+\mathbf{K}_{yz}\frac{\partial\mathbf{u}}{\partial y}+\mathbf{K}_{zz}\mathbf{u}+\mathbf{M}\frac{\partial^2\mathbf{u}}{\partial t^2}=\mathbf{f}^{\mathrm{i}}+\mathbf{f} \tag{1}$$

where $\mathbf{u}$ is the nodal displacement vector, $\mathbf{K}$ is the element stiffness matrix, and $\mathbf{M}$ represents the element mass matrix. In Eq. (1), $\mathbf{f}$ and $\mathbf{f}^{\mathrm{i}}$ are the external and internal force vectors, respectively. Note that a set of orthotropic material properties can be considered when the stiffness matrices are calculated. For a multi-element system, each local element matrix can be assembled into a global matrix. For compact notation, the same symbols are used for both local and global quantities from now on. The left-hand-side terms associated with the stiffness matrices in Eq. (1) can be represented as the linear operator defined by

$$\mathbf{L}=\mathbf{K}_{xx}\frac{\partial^2}{\partial x^2}+\mathbf{K}_{xy}\frac{\partial^2}{\partial x\partial y}+\mathbf{K}_{yy}\frac{\partial^2}{\partial y^2}+\mathbf{K}_{xz}\frac{\partial}{\partial x}+\mathbf{K}_{yz}\frac{\partial}{\partial y}+\mathbf{K}_{zz}\,. \tag{2}$$

When the system is assumed to be excited at a single angular frequency of ω, the global equation of motion can be expressed as

$$\mathbf{L}\{\dot{\mathbf{u}}(\mathbf{x})\}-\omega^2\mathbf{M}\dot{\mathbf{u}}(\mathbf{x})=-j\omega\mathbf{f}(\mathbf{x}) \tag{3}$$

where $\mathbf{x}$ represents a vector in *x*-*y* plane: i.e, $\mathbf{x} = (x, y)$. Note that the internal force vector in Eq. (1) is not shown in Eq. (3) since a pair of internal forces facing each other is cancelled during global matrix assembly process.

2.2 Sound Transmission through Hybrid FE System

When a plane wave is incident on the bottom surface of a panel at a single frequency, the external force vector in Eq. (3) is represented as the combination of incident, reflected, and transmitted sound pressures: i.e.,

$$\mathbf{L}\{\dot{\mathbf{u}}(\mathbf{x})\}-\omega^2\mathbf{M}\dot{\mathbf{u}}(\mathbf{x})=-j\omega\{\mathbf{s}_1(p_{\mathrm{i}}(\mathbf{x})+p_{\mathrm{r}}(\mathbf{x}))-\mathbf{s}_{\mathrm{N}}\,p_{\mathrm{t}}(\mathbf{x})\} \tag{4}$$

where p_{i}, p_{r}, and p_{t} are the incident, reflected, and transmitted sound pressures, respectively, and $\mathbf{s}_1$ and $\mathbf{s}_{\mathrm{N}}$ represent the unit vectors normal to the bottom and top surfaces of the panel, respectively. For an infinite-size panel, a set of wave solutions can satisfy Eq. (4): i.e.,

$$\dot{\mathbf{u}}(\mathbf{x})=\hat{\dot{\mathbf{u}}}\exp(jk_x x+jk_y y)\,, \tag{5}$$

$$p_{\mathrm{i}}(\mathbf{x})=\hat{p}_{\mathrm{i}}\exp(jk_x x+jk_y y)\,, \tag{6}$$

$$p_{\mathrm{r}}(\mathbf{x}) = \hat{p}_{\mathrm{r}} \exp(jk_x x + jk_y y)\,, \tag{7}$$

and

$$p_{\mathrm{t}}(\mathbf{x}) = \hat{p}_{\mathrm{t}} \exp(jk_x x + jk_y y) \tag{8}$$

where the upper caret represents complex magnitude. After substituting Eqs. (5) to (8) into Eq. (4) and omitting the upper carets and plane wave terms represented by exponential function, the velocity vector can be calculated as

$$\dot{\mathbf{u}} = -2\,j\omega p_{\mathrm{i}} \left\{\mathbf{K} - \omega^2\mathbf{M} - j\omega(\mathbf{R}_1 + \mathbf{R}_{\mathrm{N}})\right\}^{-1} \mathbf{s}_1 \tag{9}$$

where

$$\mathbf{K} = -k_x^2\mathbf{K}_{xx} - k_x k_y\mathbf{K}_{xy} - k_y^2\mathbf{K}_{yy} + jk_x\mathbf{K}_{xz} + jk_y\mathbf{K}_{yz} + \mathbf{K}_{zz}\,, \tag{10}$$

$$Z_{\mathrm{f}} = \rho_0 c_0 \frac{k}{k_z} = \rho_0 c_0 \frac{k}{\sqrt{k - k_x^2 - k_y^2}}\,, \tag{11}$$

$$\mathbf{R}_1 = Z_{\mathrm{f}}\mathbf{s}_1\mathbf{s}_1^{\mathrm{T}}\,, \tag{12}$$

and

$$\mathbf{R}_{\mathrm{N}} = Z_{\mathrm{f}}\mathbf{s}_{\mathrm{N}}\mathbf{s}_{\mathrm{N}}^{\mathrm{T}}\,. \tag{13}$$

Note that the last term including $\mathbf{R}_1$ and $\mathbf{R}_{\mathrm{N}}$ in Eqs. (12) and (13) represent acoustic loading on the panel. Note also that the stiffness matrix, **K** includes both real and imaginary parts regardless of the presence of damping (see Eq. (10)). Thus, spatially decaying waves can be propagating though the panels even though there is no sound radiation or structural damping. When plane wave is incident at the angles of ϕ and θ, intensity transmission coefficient is represented as a function of ϕ and θ: i.e.,

$$\tau(\phi,\theta) = \frac{|\,p_{\mathrm{t}}\,|^2}{|\,p_{\mathrm{i}}\,|^2} = \frac{|\,Z_{\mathrm{f}}\,s_{\mathrm{N}}^{\mathrm{T}}\dot{\mathbf{u}}\,|^2}{|\,p_{\mathrm{i}}\,|^2}\,. \tag{14}$$

By integrating the intensity transmission coefficient with respect to ϕ and θ, diffuse field transmission coefficient can be calculated [3]: i.e.,

$$\tau_{\mathrm{d}} = \frac{\int_0^{2\pi}\int_0^{\pi/2}\tau(\phi,\theta)\cos(\phi)\sin(\phi)d\phi d\theta}{\int_0^{2\pi}\int_0^{\pi/2}\cos(\phi)\sin(\phi)d\phi d\theta} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2}\tau(\phi,\theta)\sin(2\phi)d\phi d\theta \tag{15}$$

Then transmission loss is calculated by using

$$\mathrm{TL} = 10\log_{10}\left(\frac{1}{\tau_{\mathrm{d}}}\right). \tag{16}$$

Although the integral interval of the incidence angle θ in Eq. (15) is represented from 0° to 90°, it is limited to be θ_{max} = 78° (i.e., θ = 0° to 78°) to consider field incidence effects [3] in the following results.

3 TL RESULTS

The hybrid FE models shown in this article consist of 10 or fewer hybrid elements. As a result, it usually takes 1-2 minutes to analyze each of the following TL cases with a computer with 3.2 GHz dual Intel Xeon processors and 4 GB RAM (it is just for computational time excluding the time required for modeling). Note that it is usually take 1-2 hours to analyze a similar problem by using full, 3-D FEM/BEM models. Note also that the maximum frequency for full, 3-D FEM/BEM analysis is much lower than that of hybrid FE analysis.

Table 1: Material properties of aluminum panel

Air	Density (ρ_0)	1.21 kg/m^3
	Speed of sound (c_0)	343 m/s
Aluminum Panel (Isotropic Material)	Thickness (d)	0.005 m
	Young's modulus (E)	7.1×10^{10} Pa
	Poisson's ration (ν)	0.3292
	Density (ρ)	2700 kg/m^3
	Structural loss factor (η)	0.03

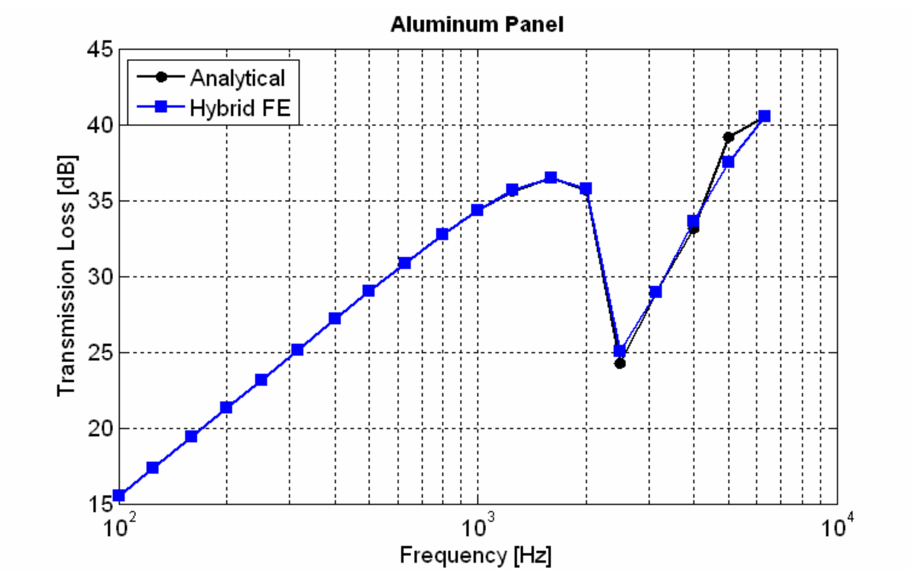

Figure 2: TL results of aluminum panel.

3.1 Aluminum Panel

As an example, 5 mm aluminum panel is analyzed by using hybrid FE method. Table 1 shows the material properties of the aluminum panel. Figure 2 shows the analytical and hybrid FE TL results. It is shown that the hybrid FE results well agree with the analytical results.

3.2 Aluminum-Foam-Aluminum Sandwich Panel

The material properties and experimental TL data of an aluminum-foam-aluminum sandwich panel are provided courtesy of the Purdue University. Table 2 shows the material properties and the TL results are shown in Fig. 3. It is observed that the predicted TL results agree well with the measured TL results although there are some discrepancies in both low and high frequency regions (e.g., below 200 Hz and above 2 kHz). In the low frequency region, the discrepancy may be caused by the finite size effects: note that an infinite-size panel model is used for the hybrid FE analysis while the experimental panel is a finite-size one.

Since the hybrid FE model uses only elastic properties for the foam core (not poroelastic properties), it may not properly represent the foam core in the high frequency region, which results in the TL difference between the predicted and measured TL results in this frequency regions. In the future, the porous effects will be considered by using poroelastic elements.

Table 2: Material properties of aluminum-foam-aluminum sandwich panel (courtesy of Purdue University)

Aluminum Face Sheet (Isotropic Material)	Structural loss factor (η)	0.10
	Other properties are the same as those given in Table 1.	
Wiltec Foam (Isotropic Material)	Thickness (d)	0.0254 m
	Density (ρ_0)	7.64 kg/m^3
	Young's Modulus (E)	2.53×10^5 Pa
	Poisson's ration (ν)	0.42
	Structural loss factor (η)	0.2
	Flow Resistivity	12180 MKS Rayls/m
	Tortuosity	1.5
	Porosity	0.99

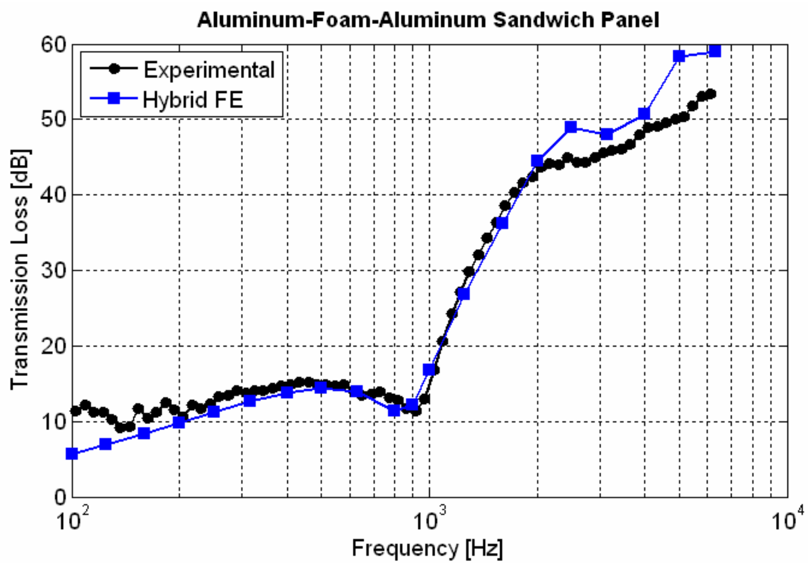

Figure 3: TL results of aluminum-foam-aluminum sandwich panel (experimental data are presented courtesy of Purdue University).

3.3 Honeycomb Sandwich Panels

The material properties of two honeycomb sandwich panels are listed in Table 3. The hybrid FE models consist of two elements for each skin and six elements for core. The TL results are compared with experimental results in Figs. 4 and 5. Below 300 Hz (500 Hz for Configuration 2), the measured TL values are higher than the predicted values due to boundary effects and imperfect reverberant conditions for Configuration 1: e.g., the maximum TL difference is approximately 7 dB for Configuration 1 (6 dB for Configuration 2). In this frequency range, however, the hybrid FE TL results well agree with the Mass Law. Note that the cut-off frequency of the reverberant room is approximately 250 Hz: i.e., below the cut-off frequency, the incident sound field of the reverberant room can not be assumed to be reverberant. In the high frequency region, the hybrid FE and experimental results well agree to each other within the difference of 1 dB for Configuration 1. For Configuration 2, these two results well agree to each other up to 4 kHz. In the frequency range form 4 kHz to 6.4 kHz, the discrepancy between the hybrid FE and experimental results may be caused by inaccurate TL measurement. It may also be caused by inaccurate prediction or measurement of material properties used for the FE model.

Table 3: Material properties of honeycomb sandwich panels

		Configuration 1	Configuration 2
Face Sheet (Isotropic Material)	Thickness (d)	5.842×10^{-4} m	4.572×10^{-4} m
	Density (ρ)	1716 kg/m^3	1778 kg/m^3
	Young's Modulus (E)	6.128×10^{10} Pa	1.523×10^{10} Pa
	Poisson's Ratio (ν)	0.143	0.142
	Loss factor (η)	0.05	0.05
Nomex Core (Orthotropic Material)	Thickness (d)	0.9017×10^{-2} m	1.905×10^{-2} m
	Density (ρ)	128.1 kg/m^3	48.1 kg/m^3
	Young's Modulus (E_{xx})	6.895×10^{5} Pa	6.895×10^{5} Pa
	Young's Modulus (E_{yy})	6.895×10^{5} Pa	6.895×10^{5} Pa
	Young's Modulus (E_{zz})	5.792×10^{8} Pa	1.310×10^{8} Pa
	Shear Modulus (G_{yz})	7.033×10^{7} Pa	2.550×10^{7} Pa
	Shear Modulus (G_{zx})	1.570×10^{8} Pa	4.900×10^{7} Pa
	Shear Modulus (G_{xy})	6.985×10^{5} Pa	6.985×10^{5} Pa
	Poisson's Ratio (ν_{yz})	0.01	0.01
	Poisson's Ratio (ν_{zx})	0.01	0.01
	Poisson's Ratio (ν_{xy})	0.50	0.50
	Loss factor (η)	0.05	0.05

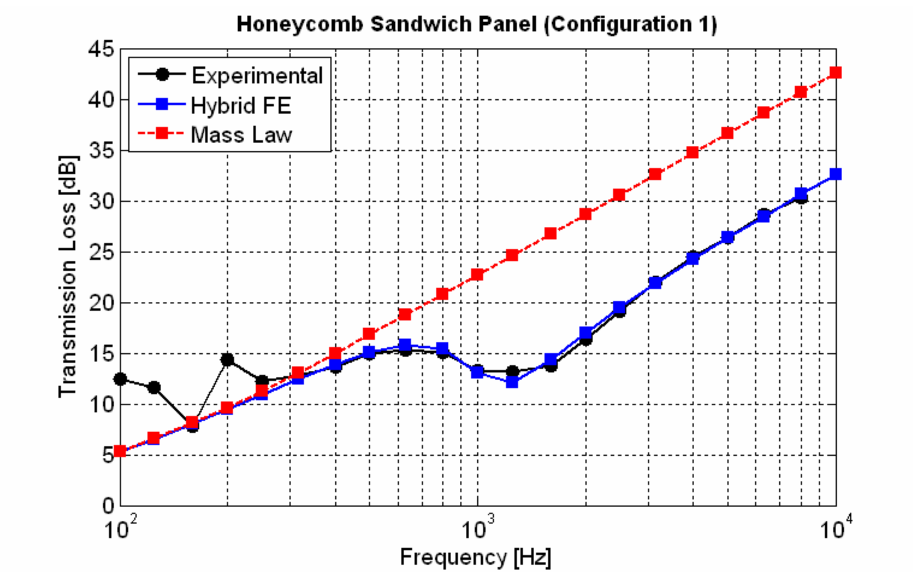

Figure 4: TL results of honeycomb sandwich panel (Configuration 1)

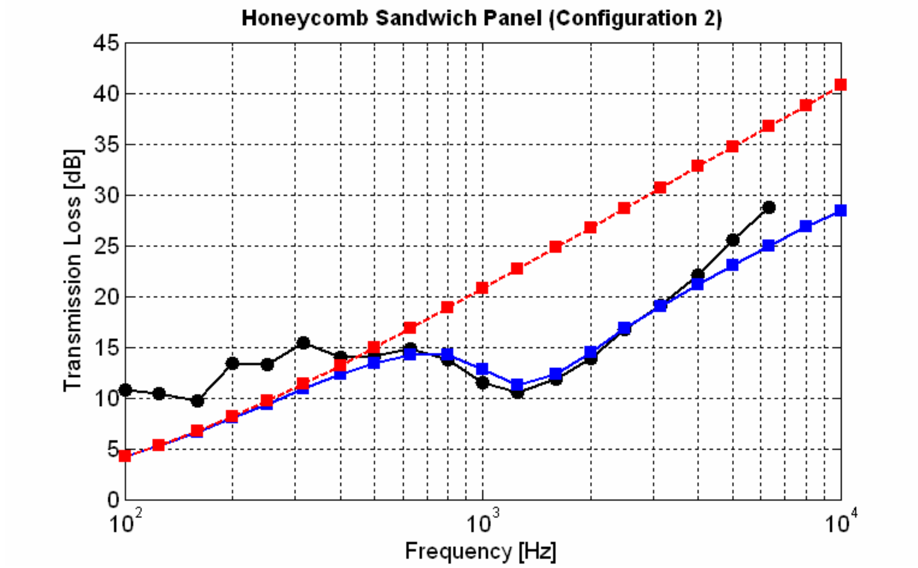

Figure 5: TL results of honeycomb sandwich panel (Configuration 2).

4 TL SENSITIVITY

Figure 6 shows the TL sensitivities of the honeycomb sandwich panel (Configuration 1) with respect to its material properties. In the low frequency region (i.e., below the critical frequency around 1.1 kHz), the TL variation is the most sensitive to the skin and core density variations while in the high frequency region, it is the most sensitive to the core shear moduli and core damping variations. Thus, the low frequency region may be referred to as mass controlled region while the high frequency region as core shear stiffness controlled region. In the critical frequency region (i.e., valley-shape TL region around 1.1 kHz in Fig. 4), the variations of all material properties except the core Young's modulus in the z-direction are important in terms of the TL sensitivities: note that the skin Young's modulus is the most sensitive to the TL variation in this critical frequency region.

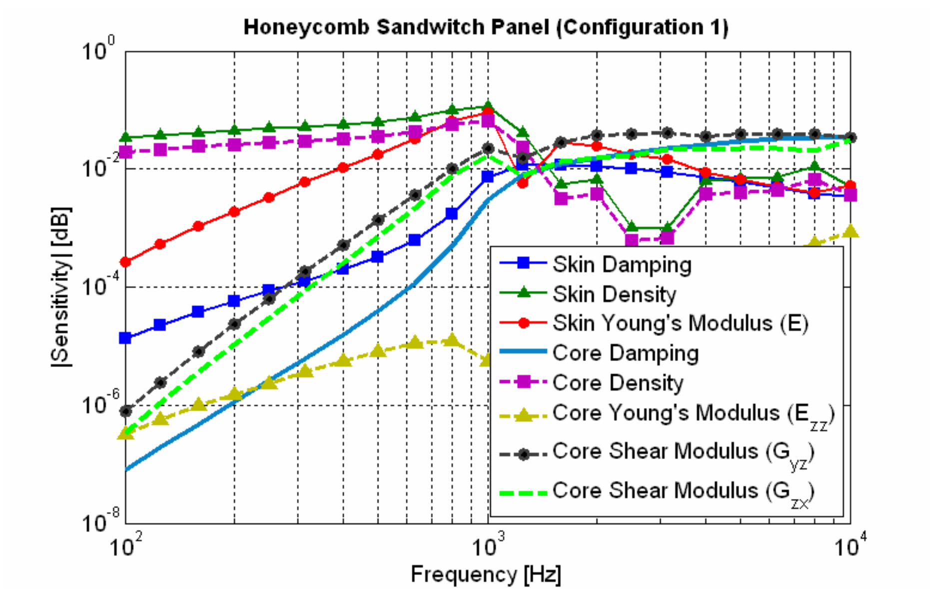

Figure 6: TL sensitivities with respect to material properties of honeycomb sandwich panel (Configuration 1).

5 WAVE PROPAGATION CHARACTERISTICS

The contour plot in Fig. 7 represents the dispersion relations of the honeycomb sandwich panel (Configuration 1) calculated by using the hybrid FE model. Note that the dispersion relations are calculated based on the vibrational response of the panel in vacuum (i.e., when there is no sound radiation). Note that in Fig. 7, the dark red represents the highest vibrational response and the dark blue represents the lowest vibrational response. On the top of the contour plot, the dispersion curves of sound wave (in air), flexural wave, and core shear waves are overlaid in straight lines. In Fig. 7, the sound and core shear waves have constant wave speeds: i.e., sound wave speed is 343m/s and core shear wave speeds are 476m/s and 711m/s. Note that the core shear wave speed is calculated by using

$$c_{\text{c.s.}} = \sqrt{\frac{G}{m}} \tag{17}$$

where G is the core shear modulus (i.e., G_{zx} or G_{yz} in Table 3) and m is the mass per unit area. For the honeycomb sandwich panel presented in Fig. 7, the sound wave is slower than shear waves, which is generally true for most Nomex honeycomb sandwich panels. Note that the core shear wave in the *x*-direction is faster than that in the *y*-direction since the core shear moduli are different in both principle directions (see the core shear moduli in Table 3). Note also that in the low frequency region, the peak vibrational response asymptotically converges to flexural wave (as frequency decreases): in the high frequency region, there are two vibrational peaks, each peaks asymptotically converge on each core shear waves (as frequency increases). In the mid frequency region, the vibrational response is in transition from flexural wave to core shear waves. The sound wave is coincident with the vibrational peak in this frequency region. As a result, the TL valley appears approximately at 1.1 kHz (see Fig. 4).

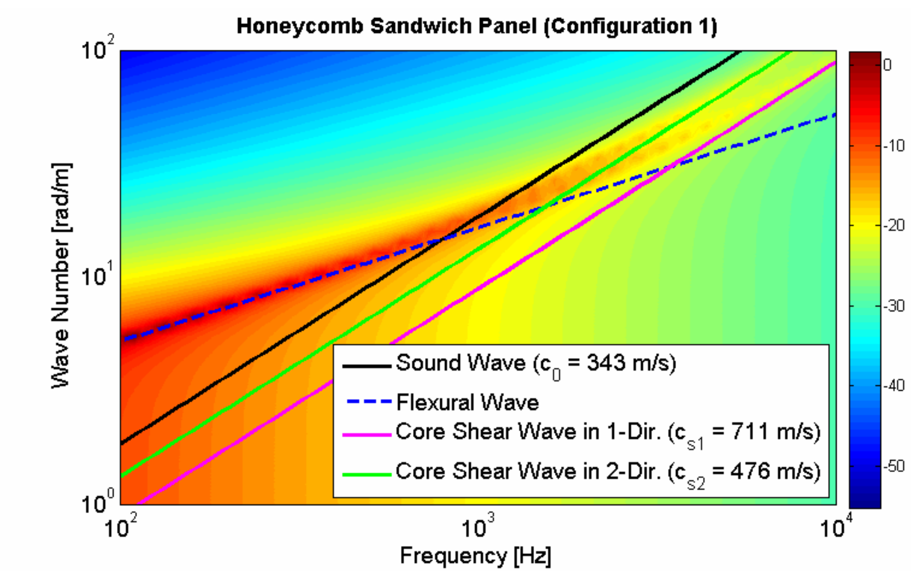

Figure 7: Dispersion relation of honeycomb sandwich panel (Configuration 1).

6 CONCLUSIONS

In this article, the hybrid analytical/one-dimensional FE method is described that can be used to analyze sound transmission characteristics of multi-layered, composite panels of which each layer is represented by orthotropic material properties. By comparing with analytical solution as well as experimental results, it is shown that the hybrid FE method can be successfully used to analyze various panels. It is also shown that the proposed method can be extended to study the effects of design variable variations on sound transmission characteristics. Through the dispersion relations of the honeycomb sandwich panel, it is observed that in low frequency region, flexural wave dominates sound transmission behaviors of the panel: while in high frequency region, core shear wave mainly affects sound transmission characteristics. Finally, the coincidence phenomenon is observed at a frequency band where the sound wave is matching with structural wave in the wave number/frequency domain. In the honeycomb panel considered in this article, the coincident phenomenon is observed at the transition region from flexural waves to core shear waves.

7 FUTURE WORK

In the future, a hybrid FE formulation for curved panels will be developed, which makes it possible for the hybrid FE method to be applied to identify the sound transmission characteristics of various curved shell structures such as airplane fuselage sections. Poroelastic and acoustic elements will be also included, allowing the hybrid FE analysis to be applicable to the porous materials. Finally, the finite size effects will be considered by using modal solutions in the plane directions.

8 ACKNOWLEDGEMENTS

The author would like to thank Evan Davis and Andrew Fung at the Boeing Company for their comments on this work. He is especially grateful to David Reed for supporting this work as a manager. He also greatly appreciates Prof. Stuart Bolton at Purdue University for his invaluable comments on this work and for providing aluminum-foam-aluminum sandwich panel data.

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