Improved Statistically Optimal Nearfield Acoustical Holography in subsonically moving fluid medium

Yong-Joe Kim*, Yaying Niu

Acoustics and Signal Processing Laboratory, Department of Mechanical Engineering, Texas A&M University, 3123 TAMU, College Station, TX 77843-3123, USA

Abstract

Statistically Optimal Nearfield Acoustical Holography (SONAH) can be used to reconstruct three-dimensional sound fields by projecting two-dimensional data measured on a “small” aperture that partially covers a composite sound source in a “static” fluid medium. Here, an improved SONAH procedure is proposed that includes the mean flow effects of a moving fluid medium while the sound source and receivers are stationary. The backward projection performance of the proposed procedure is further improved by using a wavenumber filter to suppress subsonic noise components. Through numerical simulations at Mach 0.6, it is shown that the improved procedure can accurately reconstruct sound source locations and radiation patterns: e.g., the spatially averaged reconstruction errors of the conventional and improved SONAH procedures are 15.40 dB and 0.19 dB, respectively, for a monopole simulation and 21.60 dB and 0.19 dB for an infinite-size panel. The wavenumber filter further reduces spatial noise, e.g., decreasing the reconstruction error from 1.73 dB to 0.19 dB for the panel simulation. An existing data measured in a wind tunnel operating at Mach 0.12 is reused for the validation. The locations and radiation patterns of the two loudspeakers are successfully identified from the sound fields reconstructed by using the proposed SONAH procedure.

1. Introduction

Nearfield Acoustical Holography (NAH) introduced by Williams et al. [1–3] is a powerful acoustical visualization procedure to reconstruct sound pressure, particle velocity, and sound intensity fields by projecting sound pressure data measured on a two-dimensional (2-D) measurement surface (i.e., hologram surface) into a three-dimensional (3-D) space.

When a hologram aperture is not sufficiently large to cover a composite sound source completely, the conventional NAH procedure suffers from significant truncation errors, since it is based on the assumption that the hologram surface is infinite in size [1–3]. Then, “patch” NAH procedures can be applied to the data measured by using a “small” microphone array to reconstruct 3-D sound fields properly [4–9]. The size of a measurement aperture for the conventional NAH procedure is defined in Ref. [10]. A measurement aperture that is smaller than this measurement aperture is referred to as the “small” measurement aperture where sound pressure level along the edge of the aperture is larger than background noise level resulting in significant truncation errors.

Among the various patch NAH algorithms [4–9], the planar Statistically Optimal Nearfield Acoustical Holography (SONAH) introduced by Hald [4,5] has the advantage of fast computation. It can be also modified to reconstruct sound

* Corresponding author. Tel.: +1 979 845 9779; fax: +1 979 845 3081.
E-mail addresses: joekim@tamu.edu (Y.-J. Kim), y-niu@tamu.edu (Y. Niu).

0022-460X/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jsv.2012.03.028
In this SONAH procedure, reconstructed 3-D sound fields are directly calculated in a spatial domain by using acoustical transfer function matrices. The Tikhonov regularization technique [11] is applied to regularize the matrices of plane wave functions when calculating the inverses of these matrices during the backward projections. Although the Tikhonov regularization procedure suppresses subsonic noise components outside of a radiation circle to some extent, its noise reduction performance is less effective than a wavenumber filter [11]. Therefore, it is here proposed that a wavenumber filter is applied to improve the performance of the SONAH procedure.

The conventional NAH [1–3] and SONAH [4–6] procedures are based on the sound pressure measurements in a “static” fluid medium: i.e., the mean flow effects of a convective fluid medium are not considered in these procedures. However, many NAH measurements require considering the mean flow effects: e.g., a jet noise measurement made by using a microphone array attached on the fuselage surface of a cruising aircraft and a tire noise measurement made by using a microphone array installed on a high-speed ground vehicle. These two measurements are equivalent to the case of a convective fluid medium while sound sources and receivers are not in motion.

Ruhala et al. [12] and Kwon et al. [13] presented improved NAH procedures including the mean flow effects of a subsonically convective fluid medium. The NAH procedure proposed by Ruhala et al. limits to a moving fluid medium at a low Mach number (e.g., |M| < 0.2, where M represents the Mach number) [12]. Kwon, Niu, and Kim introduced the improved NAH procedure that can be applied to a moving fluid medium at any subsonic speed (i.e., |M| < 1.0) [13]. A NAH procedure referred to as the “Moving Frame Technique” was proposed by Kwon, Park, and Kim [14–17] that can be used to reconstruct 3-D sound fields from sound pressure data measured using a stationary microphone array in a stationary fluid medium while a narrow-band source is in a constant, translational motion.

In this paper, an improved SONAH procedure is proposed that includes the mean flow effects of a subsonically convective fluid medium and a “convective” wavenumber filter. It can be then used to reconstruct sound fields by projecting the 2-D sound pressure data measured on a small measurement aperture in the subsonically convective fluid medium while the sound sources and receivers are not in motion. The proposed SONAH approach is based on the NAH theory in a convective fluid medium at |M| < 1.0 described in Ref. [13]. Thus, it can be applied to the cases of any uniformly convective fluid media at |M| < 1.0. Compared to the Moving Frame Technique [14–17] where the sound pressure data of a moving source is measured using a stationary microphone array in a stationary fluid medium, the improved SONAH approach considers the effects of a convective fluid medium between a composite source and a microphone array that are stationary. Since the Doppler shift of a single frequency component caused by the convective fluid medium does not affect other frequency component in the proposed procedure, it has no limitation on the frequency characteristics of the sources.
In addition, the proposed SONAH procedure is derived from a “generalized” NAH description. Analogous to the conventional NAH procedure based on the Discrete Fourier Transform (DFT), the generalized NAH description is “implicitly” based on a Regressive Discrete Fourier Series (RDFS) [18] in that the number of measurement points can be smaller than that of wavenumber points to minimize edge truncation effects. However, the “explicit” calculation of wavenumber spectra is not required in the generalized NAH procedure unlike the conventional NAH procedure. It is shown that the conventional planar NAH and SONAH projections are the two special cases of the generalized NAH description.

When there is a stationary body submerged in a uniformly moving fluid medium, the noise generated from the body is deflected as it propagates through the boundary layer around the body. When the boundary layer is “turbulent”, flow noise is also generated within the boundary layer. The theory presented in this article is however limited to the uniform flow region outside of the boundary layer. The boundary layer effects including viscosity and compressibility are thus ignored in the proposed SONAH procedure.

In order to validate the proposed SONAH procedure in a subsonically convective fluid medium, numerical simulations using two monopoles and an infinite-size panel are conducted where the airflow speed is set to \( |M| = 0.6 \). In addition, an existing experimental data that is measured by using two loudspeakers in a wind tunnel operating at \( |M| = 0.12 \) and published in Ref. [13] is reused for the validation. The conventional planar NAH and SONAH procedures are also used for sound field reconstructions with the purpose of comparison to the improved SONAH approach. The Cholesky Decomposition (CD) based partial field decomposition procedure [19,20] and source nonstationary compensation procedure [20] are applied to obtain incoherent partial sound pressure fields on the hologram surface in the monopole simulation and experiment cases. In order to enhance the reconstruction results of the experimental data, background noise and flow-induced microphone noise are reduced by removing the noise-associated singular values after applying the CD procedure to the measured reference cross-spectra.

2. Theory

All acoustical properties in this paper are represented at an angular frequency of \( \omega \). Thus, unless stated otherwise, the frequency dependency is implicitly assumed in this paper.

2.1. Wave propagation in subsonically convective fluid medium

Consider a coordinate system \((\chi, \psi, \zeta)\) defined in a convective fluid medium at a subsonic and uniform velocity. This coordinate system is corresponding to a Cartesian coordinate system \((x, y, z)\) in a static fluid medium. When the fluid medium is convective at a constant subsonic speed, \( U \) in the \( \chi \)-direction, while the sound sources and receivers are not in motion, the wavenumber in the \( \zeta \)-direction satisfies the following relation with the \( \chi \) - and \( \psi \)-direction wavenumbers [13]: i.e.,

\[
k_\zeta = \begin{cases} 
\sqrt{k^2 - (1-M^2)k_\chi^2 - 2k Mk_\psi - k_\psi^2} & \text{if } (1-M^2)k_\chi^2 + 2k Mk_\psi + k_\psi^2 \leq k^2 \smallskip \\
iv(1-M^2)k_\chi^2 + 2k Mk_\psi + k_\psi^2 - k^2 & \text{otherwise}
\end{cases},
\]

(1)

where \( M \) is the Mach number, \( M = U/c_0 \), \( k \) is the acoustic wavenumber, \( k = \omega/c_0 \), and \( c_0 \) is the speed of sound. When \( M = 0 \), Eq. (1) becomes a classical wavenumber relation in the static fluid medium [3]. From the convective Euler’s equation, the relation between sound pressure and particle velocity in the \( j \)-direction is represented in the \((k_\chi, k_\psi)\) wavenumber domain as [13]

\[
V_j = \frac{k_j}{\rho_0 c_0 (k-Mk_j)} P.
\]

(2)

where \( j = \chi, \psi, \text{or } \zeta \), and \( \rho_0 \) is the ambient density of the fluid medium. In Eq. (2), \( V_j \) and \( P \) are the \( j \)-direction velocity spectrum and sound pressure spectrum, respectively. The mapping function in the wavenumber domain between the static and convective fluid medium cases is defined as [13]

\[
(k_\chi, k_\psi) = (k(k_\chi + a)/r_1, kk_\psi/r_2),
\]

(3)

where

\[
a = \frac{kM}{1-M^2},
\]

(4)

\[
r_1 = \frac{k}{1-M^2},
\]

(5)

and

\[
r_2 = \frac{k}{\sqrt{1-M^2}}.
\]

(6)
Eqs. (3)–(6) are derived from the comparison between the radiation “ellipse” in the convective fluid medium and the classical radiation circle in the static fluid medium [13].

2.2. Generalized NAH description in convective fluid medium

In a spatial domain, the relation between measured and reconstructed sound pressures has the general form [4,5] of

$$ p_r = H_p p_h, $$

(7)

where $p_r$ is the vector of the reconstructed sound pressures at $N$ reconstruction locations on a reconstruction surface, $p_h$ is the vector of the measured sound pressures at $M$ measurement points on a hologram surface, and $H_p$ is then the $N \times M$ acoustical transfer function matrix. In Eq. (7), $p_h$ is measured and $H_p$ is determined by using a NAH procedure to calculate the reconstructed sound pressure vector. Similarly, a $j$-direction particle velocity vector can be reconstructed from the measured sound pressure vector by using the following acoustical transfer function matrix [4,5]: i.e.,

$$ v_j = H_j p_h, $$

(8)

where $j = \chi, \psi, \text{or} \zeta$. The NAH reconstructions in Eqs. (7) and (8) are represented in the coordinate system, $(\chi, \psi, \zeta)$ defined in the convective fluid medium, while the same equations in Refs. [4,5] are represented in a static fluid medium. In order to derive the expressions of $H_p$ and $H_j$, consider the inverse spatial Fourier Transforms of the sound pressure and $j$-direction velocity spectra: i.e.,

$$ p(\chi, \psi, \zeta) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [P(k_\chi, k_\psi)e^{ikk}e^{ik_\chi \chi + k_\psi \psi}] dk_\chi dk_\psi, $$

(9)

$$ v_j(\chi, \psi, \zeta) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{k_j}{\rho_0 c_0 (k - Mk_{\chi j})} P(k_\chi, k_\psi)e^{ikk}e^{ik_\chi \chi + k_\psi \psi}] dk_\chi dk_\psi, $$

(10)

where $P(k_\chi, k_\psi)$ is the sound pressure spectrum on the source surface in the wavenumber domain, $(k_\chi, k_\psi)$. Eq. (2) is substituted into Eq. (9) to result in Eq. (10). The exponential function with the $\zeta$-direction wavenumber, $\exp(ikk_{\chi \zeta})$ in Eq. (9) represents a sound pressure propagator that relates the sound pressure spectra on the source surface ($\zeta = 0$) and the surface at $\zeta$. The inverse spatial Fourier Transforms in Eqs. (9) and (10) can be discretized as

$$ p(\chi, \psi, \zeta) \approx \sum_j P(k_{j\chi}, k_{j\psi})e^{ikk}e^{ik_\chi \chi + k_\psi \psi + k_{j\chi}}, $$

(11)

$$ v_j(\chi, \psi, \zeta) \approx \sum_j \frac{k_{j\chi}}{\rho_0 c_0 (k - Mk_{\chi j})} P(k_{j\chi}, k_{j\psi})e^{ikk}e^{ik_\chi \chi + k_\psi \psi + k_{j\chi}}, $$

(12)

where $k_{j\chi}, k_{j\psi}$, and $k_{j\chi}$ represent discretized wavenumbers. In matrix forms, the discrete inverse spatial Fourier Transforms are represented as

$$ p_h = \Phi_p P, $$

(13)

$$ p_r = \Phi_p P, $$

(14)

and

$$ v_j = \Phi_j P, $$

(15)

where $P$ is the vector of the sound pressure spectrum on the source surface. The $(m,l)$ element in the plane wave matrix, $\Phi_h$, in Eq. (13) is defined as

$$ [\Phi_h]_{m,l} = e^{ik_{l\chi}z_m + k_{l\psi} \psi_m + k_{l\chi \eta} \eta_m}. $$

(16)

Similarly, the plane wave matrices in Eqs. (14) and (15) can be written as

$$ [\Phi_p]_{m,l} = e^{ik_{l\chi}z_m + k_{l\psi} \psi_m + k_{l\chi \eta} \eta_m}, $$

(17)

$$ [\Phi_j]_{m,l} = \frac{k_{j\chi}}{\rho_0 c_0 (k - Mk_{\chi j})} [\Phi_j]_{m,l}. $$

(18)

By applying Eqs. (13)–(15) into Eqs. (7) and (8), the following two equations can be derived as

$$ \Phi_p P = H_p \Phi_h P, $$

(19)

$$ \Phi_j P = H_j \Phi_h P. $$

(20)

From Eqs. (19) and (20), the least square solutions of the two acoustical transfer function matrices can be then written as

$$ H_p = \Phi_p P H_h \Phi_p P H_h \Phi_p P H_h + \delta^2 I)^{-1}, $$

(21)
\[ H_i = \Phi_i PP_i^H \Phi_h^H (\Phi_i PP_i^H \Phi_h^H + \vartheta^2 I)^{-1}, \]  
where the superscript, H denotes the Hermitian (i.e., conjugate transpose) of a complex matrix. The Tikhonov regularization is applied in Eqs. (21) and (22). As presented in Ref. [5], the regularization parameter is given as \[ \vartheta^2 = \left( 1 + \frac{1}{2(k_c h)^2} \right) 10^{-\text{SNR}/10}, \]

where \( z \) is the hologram height and SNR represents the signal to noise ratio.

In the conventional planar NAH procedure [1–3], the reconstruction points on a source surface are defined on regular mesh points that can be obtained by linearly translating the hologram points. Additionally, wavenumber spectrum is obtained by using the spatial Fast Fourier Transform (FFT). Therefore, \( \Phi_h \) is a square matrix and can be directly inversed without using the least square solution procedure shown in Eqs. (21) and (22); i.e., the inverse of \( \Phi_h \) represents a backward NAH projection from the measurement surface to source surface and \( \Phi_r \) is a forward NAH projection from the source surface to reconstruction surfaces. Therefore, the conventional NAH procedure is one of the special cases of Eqs. (7), (8), (21), and (22). However, \( \Phi \) is not a square matrix in general, e.g., when the number of the discrete wavenumbers is chosen to be larger than the number of the measurement points. In addition, the sound pressure spectrum, \( P \) cannot be obtained accurately on a small measurement aperture: i.e., the small measurement aperture results in the sound pressure spectrum in low spectral resolutions.

When setting \( PP_i^H = I \) in Eqs. (21) and (22), the resulting equations become identical to the conventional SONAH expressions as Hald presented in Refs. [4,5]. Therefore, the NAH description presented in this section can be considered as a generalized SONAH description. The condition of \( PP_i^H = I \) indicates implicitly that all of the wavenumber components contribute equally to the NAH projection procedure: e.g., the measured sound pressure spectrum is used for backward projections without any wavenumber filtering process. However, it is necessary to reduce the subsonic components during the backward projection procedures, since the noise components in this subsonic region are amplified exponentially during the backward projection procedures [1–3]. Although the matrix regularization can reduce the subsonic noise components to some extent, it is proposed to use a wavenumber filter to effectively suppress the subsonic noise components before performing the backward projections in SONAH procedures.

### 2.3. Improved SONAH procedure in convective fluid medium

A wavenumber filter in the convective fluid medium can be obtained by applying the mapping relation of Eq. (3) to the wavenumber filter in the static fluid medium presented in Refs. [3,21]: i.e., when \( k \leq k_c \),

\[ W(k_x,k_y) = \begin{cases} 
1 - \exp[(k_x/k_c - 1)/\alpha]/2 & \text{if } k_x \leq k_c \\
\exp[(k_x/k_c - 1)/\alpha]/2 & \text{otherwise}
\end{cases}, \]

otherwise,

\[ W(k_x,k_y) = \begin{cases} 
1 & \text{if } k_x \leq k \\
0 & \text{otherwise}
\end{cases}, \]  

where \( k_c = k \sqrt{(k_x + a)^2/r_1^2 + k_y^2/r_2^2}. \)

In Eq. (24), \( \alpha \) and \( k_c \) are the filter slope and cut-off wavenumber, respectively. In general, \( \alpha \) and \( k_c \) should be determined from the dynamic range of a measurement system, SNR, hologram height, \( \zeta_h \), and sound pressure spectral components [3]. In the current research, \( \alpha \) is set to 0.2 and \( k_c \) is determined by the following equation: i.e.,

\[ k_c = 2\pi/(3\zeta_h), \]  

where \( \zeta_h \) is hologram height [21].

In order to effectively suppress the subsonic noise components outside of the radiation ellipse, it is here proposed to use the convective wavenumber filter in Eqs. (24)–(27) for the calculation of the acoustical transfer function matrices in Eqs. (21) and (22): i.e.,

\[ PP_i^H = \text{diag} \left[ W^2(k_{x1},k_{y1}) \quad W^2(k_{x2},k_{y2}) \quad \cdots \right]^T. \]  

Sound pressure and particle velocity fields can be reconstructed by calculating the acoustical transfer function matrices from Eqs. (21), (22), and (28) and substituting the calculated acoustical transfer function matrices into Eqs. (7) and (8). Acoustical intensity fields can be also reconstructed by multiplying the reconstructed sound pressure and particle velocity fields.

Arbitrarily located measurement points can be used as the measurement points in the proposed SONAH procedure. When regularly meshed points are chosen as the measurement points, the sampling spaces in the \( \chi \)- and \( \psi \)-directions are
represented as [13]

\[ \Delta_x = \frac{c_0(1 - |M|)}{2sf_{\text{max}}}, \]  

(29)

\[ \Delta_\phi = \frac{c_0 \sqrt{1 - M^2}}{2sf_{\text{max}}}, \]  

(30)

In Eqs. (29) and (30), \( s \) is the over-sampling rate \( (s \geq 1) \) and set to \( s = 2 \) in the current research.

3. Numerical simulations and experiment

3.1. Monopole simulation

A monopole simulation in a convective fluid medium is conducted to validate the proposed SONAH approach. For the purpose of describing the sound pressure field radiated from a monopole in the convective fluid medium, consider the case of the convective fluid medium that is moving at a subsonic and uniform speed of \( U \) in the \( \chi \)-direction, while the monopole source and microphone array are fixed at their positions. The sound pressure field radiated from the monopole is then represented as [13]

\[ p(r) = \frac{Q(\omega)e^{i\mathbf{r}\cdot\mathbf{f}}}{R_1}, \]  

(31)

where \( \mathbf{r} = \chi \mathbf{e}_\chi + \psi \mathbf{e}_\psi + \zeta \mathbf{e}_z \). In Eq. (31), \( Q(\omega) \) is the monopole strength and \( R \) and \( R_1 \) are represented as

\[ R = \frac{-M\chi + R_1}{1 - M^2}, \]  

(32)

\[ R_1 = \sqrt{\chi^2 + (1 - M^2)(\psi^2 + \zeta^2)}. \]  

(33)

The experiment shown in Fig. 1 is simulated by using the monopole sound pressure field calculated from Eqs. (31)–(33). The \( \chi \)-direction sampling space for the monopole simulation is 0.02 m, while it is set to 0.05 m in the experiment. The \( \psi \)-direction sampling space is set to 0.025 m for the monopole simulation and 0.05 m for the experiment. The hologram height is \( \zeta_h = 0.06 \) m for both the monopole simulation and experiment. In the monopole simulation, two incoherent monopoles are located at \((\chi, \psi, \zeta) = (0.3, 0.25, -0.05) \) m and \((0.5, 0.1, -0.05) \) m of which the \( \chi \) - and \( \psi \)-locations are the locations of the two loudspeakers in the experiment. The monopoles are placed at \( \zeta = -0.05 \) m in order to avoid the monopole’s singularity at \( R_1 = 0 \). The upper left monopole is located at the top edge of the microphone array to signify the edge effects. The total number of measurement points on the hologram surface is \( 11 \times 36 \) and the hologram sound pressure data is calculated for 10 s at the sampling frequency of 8192 Hz. Two reference sound pressure signals are obtained from the two points at the \( \chi \)- and \( \psi \)-locations of the two monopoles and \( \zeta = -1 \) mm. The convective speed of the fluid medium is set to \( M = -0.6 \), which is similar to a cruising speed of modern jet airplanes. The negative Mach number indicates that the airflow direction is opposite to the pre-defined positive \( \chi \)-direction.

![Fig. 1. Sketch of experimental setup.](image-url)
3.2. Simulation with infinite-size panel

A monopole has a broad wavenumber spectrum since it is a “point” source. When a panel is vibrating at one of its resonance frequencies, a single mode shape is dominantly excited. The wavenumber spectrum of this single-mode-dominant vibration response contains only narrow-band wavenumber spectral components. Therefore, it is generally much more challenging to reconstruct sound fields radiated from a monopole than a panel.

On the other hand, sound radiation from a panel can be an excellent validation case of the proposed SONAH procedure since it is easy to simulate sound pressure truncation errors at the edges of a microphone array by having its aperture smaller than the panel. Here, an infinite-size panel is considered as an extreme case where there are always truncation errors with a finite-size microphone array. The same microphone array used in the monopole simulation is used in the infinite panel simulation while it is set to scan only single scanning position at the hologram height of \( \zeta_h = 0.06 \) m. The convective speed of the fluid medium is also set to \( M = -0.6 \). It is additionally assumed that the panel's normal vibration response is superposed by the two sets of wavenumber components, \((k_x, k_y) = (50, 5)\) and \((0, 25)\) at 1.5 kHz. The normal velocity amplitudes of these two wavenumber components are set to 0.2 m/s and 0.1 m/s, respectively. Sound pressures and particle velocities in a 3-D space can be found from Eqs. (1) and (2) for the given wavenumbers and their normal velocity amplitudes.

3.3. Experimental setup

Fig. 1 shows the experimental setup including a microphone array and two loudspeakers [13]. Since the measurement aperture of this experimental setup is reduced by removing the upper two microphone rows from the experimental setup in Ref. [13], the experiment is briefly summarized in this section. The array microphones installed with wind screens (0.03 m in diameter) at the microphone ends have the diameters of one quarter inch (7 mm). The two loudspeakers driven by a two-channel power amplifier are placed at \((\chi, \psi, \zeta) = (0.3, 0.25, 0)\) m and \((0.5, 0.1, 0)\) m. Two waveform generators provide incoherent excitation signals to the two loudspeakers. For the upper left loudspeaker, the maximum sound pressure level (SPL) is approximately 98 dB on the hologram plane, while the lower right loudspeaker has the maximum SPL of 86 dB. The background flow noise level is approximately 60 dB at the measurement locations.

The microphone array size is \(6 \times 5\) and the total number of scans is 3. Thus, the total number of the measurement points is \(6 \times 15\). This scan-based measurement method is introduced by Hald [10]. The spatial sampling spaces (i.e., array microphone spaces) are 0.05 m in both the \(\chi\)- and \(\psi\)-directions and the hologram height is 0.06 m. Sound pressure data is recorded for 20 s at each scanning location at the sampling frequency of 8192 Hz. Reference signals are recorded by using six reference microphones positioned apart from the wind tunnel test section on the opposite side of the loudspeakers. The excitation signals directly measured from the waveform generators are used as two additional reference signals. Since there are two independent sound sources (i.e., two loudspeakers), at least two reference signals are needed. The background noise including the flow-induced microphone noise on the measurement surface is reduced by removing the third and higher order singular values after decomposing the cross-spectral matrix of the reference signals by using the CD-based procedure [19,20]. Table 1 shows the singular values at the reconstruction frequency of 1.5 kHz. The first and second singular values are larger than the other six singular values that are close to each other and thus the first two partial fields are investigated in this article.

The fluid flow speed is set to \(M = -0.12\) in the experiment that can be assumed to be uniform across the \(0.4 \times 0.4\) m wind tunnel test section as illustrated in Fig. 1. The assumption of the uniform velocity is based on velocity measurements across the wind tunnel test section. Similar to the numerical simulations, the airflow direction is opposite to the positive \(\chi\)-direction.

The microphone array is placed above a hard surface that can be assumed to be rigid. The distance between the microphone array and the rigid plane is one half of the microphone space, 0.025 m. In order to consider the mirroring effect of the rigid surface, the measured sound pressure data on the hologram plane is mirrored with respect to the rigid plane. Then, the total sound pressure data obtained by combining the measured sound pressure data and mirrored data is processed as that the total pressure data is measured in a free field.

3.4. Numerical simulation results

Fig. 2 shows partial sound pressure fields on the hologram plane at \(\zeta = 0.06\) m. The first sound pressure field mainly radiated from the upper left monopole source at \((0.3, 0.25, 0)\) m is shown in Fig. 2(a), while the second sound pressure field primarily radiated from the lower right monopole source at \((0.5, 0.1, 0)\) m is plotted in Fig. 2(b). Thus, each partial sound

![Table 1](image)

<table>
<thead>
<tr>
<th>SV (dB)</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>105</td>
<td>75</td>
<td>65</td>
<td>62</td>
<td>60</td>
<td>58</td>
<td>57</td>
<td>53</td>
</tr>
</tbody>
</table>
The pressure field can be associated with each of the two monopole sources. The partial sound pressure fields in Fig. 2 are obtained by using the CD-based partial field decomposition technique [19,20]. The two partial fields on the hologram plane are then used to reconstruct sound fields separately.

**Fig. 2.** Monopole simulation results: partial sound pressure fields on hologram surface at $z = 0.06$ m, $|M| = 0.6$, and $f = 1.5$ kHz: (a) First sound pressure field and (b) second sound pressure field.

**Fig. 3.** Monopole simulation results: total sound pressure fields on source surface at $z = 0$ m, $|M| = 0.6$, and $f = 1.5$ kHz: (a) Directly calculated, (b) conventional planar NAH procedure with “static” wavenumber filter, and (c) improved planar NAH procedure with “convective” wavenumber filter.

pressure field can be associated with each of the two monopole sources. The partial sound pressure fields in Fig. 2 are obtained by using the CD-based partial field decomposition technique [19,20]. The two partial fields on the hologram plane are then used to reconstruct sound fields separately.
The total sound pressure field on the source surface is obtained by superposing the two partial sound pressure fields: i.e.,

$$|p_t(\chi,\psi)| = \sqrt{|p_1(\chi,\psi)|^2 + |p_2(\chi,\psi)|^2},$$

(34)

where $|p_1|$ and $|p_2|$ are the magnitudes of the partial sound pressures and $|p_t|$ is the magnitude of the total sound pressure. The following total field results are calculated by using the same superposition procedure presented in Eq. (34). All of the following sound pressure fields are represented as sound pressure levels (SPLs) in dB scale referenced at 20 μPa.

Figs. 3 and 5 show the reconstructed, total sound pressure fields on the source surface at $\zeta = 0$ m. Here, the directly calculated, total sound pressure field shown in Fig. 3(a) serves as a baseline: i.e., the total sound pressure fields reconstructed by using various NAH procedures are compared with this baseline.

The sound pressure field reconstructed by using the conventional planar NAH procedure [1–3] with a “static” wavenumber filter is presented in Fig. 3(b). Through the comparison between the reconstruction result in Fig. 3(b) and the directly calculated sound pressure field in Fig. 3(a), it can be concluded that the conventional planar NAH procedure fails to reconstruct sound source locations and radiation patterns correctly. In particular, the source locations identified from Fig. 3(b) shift to the negative $\chi$-direction since the conventional planar NAH procedure cannot account for the mean flow effects. In addition, the $\psi$-direction location of the upper left sound source is inaccurately reconstructed at $\psi = 0.18$ m due to the small measurement aperture that cannot completely cover the upper left source located at $(0.3, 0.25, 0)$ m.

The improved planar NAH procedure [13] can consider the mean flow effects. The total sound pressure field reconstructed by using the latter procedure is shown in Fig. 3(c). When compared with the total sound pressure fields in Fig. 3(a) and (b), the result in Fig. 3(c) shows the $\chi$-direction sound source locations correctly but the $\psi$-direction location of the upper left monopole source inaccurately due to the truncation of the measured data at the top measurement edge. The reconstruction errors between the directly calculated sound pressure field (Fig. 3(a)) and the sound pressure fields reconstructed by using the two conventional planar NAH procedures (Fig. 3(b) and (c)) are presented in Fig. 4(a) and (b) where the maximum reconstruction error is approximately 25 dB. Fig. 4(c) shows the reconstruction error between the directly calculated field and the reconstructed field of the improved SONAH procedure with the

---

Fig. 4. Monopole simulation results: reconstruction errors in Eq. (35) on source surface at $\zeta = 0$ m, $|M| = 0.6$, and $f = 1.5$ kHz: (a) Conventional planar NAH procedure with “static” wavenumber filter (spatially averaged reconstruction error in Eq. (36), $e_a = 9.52$ dB), (b) improved planar NAH procedure with “convective” wavenumber filter ($e_a = 5.95$ dB), and (c) improved SONAH “with” mean flow effects and convective wavenumber filter ($e_a = 0.19$ dB).
maximum error of 0.7 dB. Here, the reconstruction error at a reconstruction point, $(x_m, \psi_n)$, is calculated as

$$e(x_m, \psi_n) = |\text{SPL}(x_m, \psi_n) - \text{SPL}_c(x_m, \psi_n)|,$$  \hspace{1cm} (35)

where $|\text{SPL}|$ is the SPL of total sound pressure reconstructed by using the planar NAH, conventional SONAH, or improved SONAH algorithms, and $|\text{SPL}_c|$ is the SPL of the directly calculated total sound pressure. A spatially averaged reconstruction error is then defined as

$$e_a = \frac{1}{N_x N_y} \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} e(x_m, \psi_n),$$  \hspace{1cm} (36)

where $N_x$ and $N_y$ are the numbers of the reconstruction points in the $x$- and $y$-direction, respectively.

Fig. 5 shows the total sound pressure fields reconstructed by using the conventional [4,5] and improved SONAH procedures. The conventional SONAH can be applied only to static fluid medium cases, and no wavenumber filter is used in this procedure. As shown in Fig. 5(a), the conventional SONAH approach completely fails to reconstruct the total sound pressure field on the source surface. When the mean flow effects are considered without the “convective” wavenumber filter, the reconstructed sound pressure field shows the correct sound source locations (see Fig. 5(b)). However, there is noticeable spatial noise (see the wavy contour lines in Fig. 5(b)). The total sound pressure field reconstructed by using the proposed SONAH procedure with the “convective” wavenumber filter is presented in Fig. 5(c) to be compared with the results reconstructed by using the other SONAH procedures in Fig. 5(a) and (b). When compared with the reconstructed, total sound pressure fields in Figs. 3(b) and (c) and 5(a) and (b), the total sound pressure field reconstructed by using the proposed SONAH procedure with the convective wavenumber filter results in the most accurate sound source locations and radiation patterns (see Fig. 5(c)) that match well with the directly calculated, total sound pressure field in Fig. 3(a) at the spatially averaged reconstruction error of 0.19 dB (see Fig. 4(c)). Through the comparison between the reconstructed total fields in Fig. 5(b) and (c), it can be also concluded that the convective wavenumber filter can effectively suppress the spatial noise. Similar to the error plots in Fig. 4, Fig. 6 shows the reconstruction errors between the directly calculated

![Fig. 5](image_url). Monopole simulation results: total sound pressure fields on source surface at $z = 0$ m, $|M| = 0.6$, and $f = 1.5$ kHz: (a) Conventional SONAH “without” mean flow effects and wavenumber filter, (b) improved SONAH with mean flow effects but “no” wavenumber filter, and (c) improved SONAH “with” mean flow effects and convective wavenumber filter.
sound pressure field (Fig. 3(a)) and the sound pressure fields reconstructed by using the SONAH algorithms (Fig. 5). The conventional “static” SONAH procedure results in the enormous reconstruction error at the maximum of approximately 35 dB (see Fig. 6(a)). By comparing Fig. 6(b) and (c), it is shown that the wavenumber filter reduces the maximum reconstruction error from 3 dB to 0.7 dB and the spatially averaged reconstruction error from 0.49 dB to 0.19 dB. Fig. 6 therefore verifies that the improved SONAH procedure with the convective wavenumber filter (see Fig. 6(c)) can reconstruct sound pressure field with the smallest reconstruction error below 0.7 dB when compared to the other two SONAH algorithms in Fig. 6(a) and (b).

Fig. 7 shows the directly calculated and reconstructed sound pressure fields on the surface of the infinite-size panel ($z = 0$ m). Compared to the directly calculated sound pressure field in Fig. 7(a), the conventional NAH procedures fail to reconstruct the sound pressure fields on the panel due to the truncation errors (see Fig. 7(b) and (c)) although their reconstructed sound fields show the traces of the two tilted stripes in the middle of the measurement aperture: see the 126 dB contour lines in Fig. 7(b) and the 136 dB line in Fig. 7(c). The spatially averaged reconstruction errors are 15.83 dB and 9.14 dB for the conventional and improved NAH procedures, respectively. The conventional SONAH without mean flow effects also significantly overestimates the sound pressure level and reconstructs the two tilted stripes as the two vertical stripes with the averaged reconstruction error of 21.60 dB (see Fig. 7(d)). When considering the mean flow effects by using the improved SONAH with no spatial wavenumber filter, the averaged reconstruction error significantly reduces to 1.73 dB as shown in Fig. 7(e). The spatially averaged reconstruction error can be further reduced to 0.19 dB by using the “convective” wavenumber filter in the proposed SONAH procedure (see Fig. 7(f)).

### 3.5. Experimental results

Fig. 8 shows the partial sound pressure fields, on the hologram plane at $\zeta = 0.06$ m, decomposed from the measured reference cross-spectral matrix (see Fig. 1 for the experimental setup where the airflow speed is set to $M = -0.12$). The first hologram partial sound pressure field, mainly radiated from the upper left loudspeaker located at (0.3, 0.25, 0) m, is presented in Fig. 8(a), while Fig. 8(b) shows the second partial sound pressure field dominantly radiated from the lower
right loudspeaker placed at (0.5, 0.1, 0) m. In Fig. 8(a), there is the small power leakage generated from the CD-based partial field decomposition procedure [19,20] (see the 89 dB contour line).

Projected from the hologram partial pressure fields by using the improved SONAH procedure, the partial sound pressure fields on the source surface at \( z = 0 \) m are reconstructed (see Figs. 9 and 10). The partial sound pressure results in Fig. 9 are obtained by using the proposed SONAH procedure without the convective wavenumber filter, while the filter is considered in the results in Fig. 10. The spatial noise shown in Fig. 9 is not present in Fig. 10 after applying the convective wavenumber filter. In particular, the spatial noises at (0.2, 0) m and (0.5, 0) m in Fig. 9 are significantly suppressed by using the convective wavenumber filter when compared with Fig. 10. Fig. 10(a) shows the first partial sound pressure field that is mainly associated with the upper left loudspeaker, although small power leakage from the lower right loudspeaker can be identified at the 92 dB contour line close to the location of the lower right loudspeaker. Fig. 10(b) shows the second partial sound pressure field dominantly radiated from the lower right loudspeaker. The magnitude of the power leakage in the first partial pressure field is larger than that of the second partial pressure field, which is caused by the imperfectly decomposed partial fields on the hologram surface (see Fig. 8) that are the inputs of the proposed SONAH algorithm. In order to reduce the power leakage, it is recommended to apply the virtual reference procedure [19].

Figs. 11 and 12 present the \( \zeta \)-direction particle velocity and active sound intensity fields on the source surface at \( \zeta = 0 \) m. Similar to the reconstructed sound pressure fields in Fig. 10, the small power leakage still exists in the first fields at the location of the lower right loudspeaker (see Figs. 11(a) and 12(a)). However, the two loudspeaker locations and their radiation patterns can be identified clearly from the reconstruction results. Through the partial field results presented in Figs. 10–12, it is shown that the improved SONAH procedure can be used to successfully reconstruct the sound fields even
when the hologram data has the significant truncation and the fluid medium is convective at the uniform and subsonic velocity.

4. Conclusions

In this paper, the improved SONAH procedure is introduced that can be used to reconstruct sound fields from sound pressure data measured using a "small" stationary microphone array placed in a fluid medium convective at a uniform and subsonic velocity. The proposed SONAH procedure is derived from the generalized NAH description. It is shown that the conventional NAH and SONAH procedures are the two special cases of the generalized NAH description. The convective wavenumber filter is also proposed to be used to effectively suppress subsonic noise components in the convective fluid medium. Through the monopole and infinite-size panel simulations where the airflow speed is set to \(|M| = 0.6\), it is shown that the conventional planar NAH procedures inaccurately reproduce the sound pressure fields on the source surface due
to the significant truncation errors even when the mean flow effects are considered in the improved planar NAH procedure proposed in Ref. [13]. In the monopole simulations, the spatially averaged reconstruction error of the conventional planar NAH procedure is 9.52 dB. When the mean flow effects are considered in the conventional planar NAH procedure, the reconstruction error decreases to 5.95 dB. Projected from the simulated sound pressure data by using the improved SONAH procedure, the sound pressure field on the source surface can be successfully reconstructed: i.e., the reconstructed, total sound pressure field matches well with the directly calculated, total sound pressure field with the spatially averaged reconstruction error of 0.49 dB. The reconstruction error further decreases to 0.19 dB when the convective wavenumber filter is applied in the proposed SONAH procedure. By applying the improved SONAH procedure to the experimental data at $|M| = 0.12$, the sound pressure, particle velocity, and active intensity fields are reconstructed correctly on the source surface to identify the two loudspeaker locations and their radiation patterns. Through the numerical simulations and

---

**Fig. 10.** Experimental results: sound pressure fields on source surface at $\zeta = 0$ m, $|M| = 0.12$, and $f = 1.5$ kHz, reconstructed by using improved SONAH procedure with mean flow effects and convective wavenumber filter: (a) First sound pressure field and (b) second sound pressure field.

**Fig. 11.** Experimental results: $\zeta$-direction particle velocity fields on source surface at $\zeta = 0$ m, $|M| = 0.12$, and $f = 1.5$ kHz, reconstructed by using improved SONAH procedure with mean flow effects and convective wavenumber filter: (a) First $\zeta$-direction particle velocity field and (b) second $\zeta$-direction particle velocity field.
experiment, it is also shown that the convective wavenumber filter used in the proposed SONAH procedure can effectively suppress the spatial noise in the reconstructed sound fields.

Acknowledgements

The authors are grateful to Dr. Dennis L. O’Neal, Associate Dean for Research and Interim Deputy Director of TEES in Texas A&M University for his support on this work. They also thank Dr. Hyu-Sang Kwon at the Korea Research Institute of Standards and Science for providing his invaluable comments for framing a research program on Nearfield Acoustical Holography and his conducting the experiment to obtain the experimental data that is published in Ref. [13] and this article.

Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jsv.2012.03.028.

References


