Ground monitoring of bottom hole assembly vibration in drill string system using Acoustic Transfer Functions and Hybrid Analytical/Two-Dimensional Finite Element Method

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When an analytical approach is used to model the vibration of a drill string system (DSS), it is difficult to include detailed modeling information such as the effects of mud or the geometries of complex pipe connections, while a full 3-D numerical approach is extremely expensive due to its large model size to model the long DSS. Here, the Acoustic Transfer Functions (ATFs) identified by using the Hybrid Analytical/Two-Dimensional Finite Element Method (2D HAFEM) are used to estimate the vibration excitation force and torque at the bottom hole assembly (BHA) of the DSS from the acoustic measurements made on the ground. Since the HAFEM uses an analytical wave solution in the axial direction of the pipe, while its response in the cross-sectional directions is represented by using a FE approximation, the HAFEM model is much more computationally efficient than conventional full 3-D models, while it can include the detailed modeling information. For the validation of the BHA vibration monitoring method, full 3-D FE analyses and experiments with a drill string system have been conducted. It is shown that the force and torque estimated by using the HAFEM-based ATF approach match well with the excitation force and torque except resonance frequencies.

1 INTRODUCTION

The vibration at the Bottom Hole Assembly (BHA) of a Drill String System (DSS) is generally monitored by using vibration transducers installed in or close to the BHA. Then, the measured signals are reduced, encoded in mud pulses, and transmitted through the mud to the ground. This mud pulse telemetry has a limited band width that makes it difficult to transmit all necessary information in a real time to set appropriate drilling strategies. In rare cases, it fails to transmit mud pulse signals to the ground due to low signal-to-noise-ratio (SNR), which makes drilling operators blind to downhole drilling conditions. Recently, the measured vibration signals

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have been directly transmitted through wires with replays, which are referred to as the “telestrings”. The latter approach can be used to overcome the aforementioned issues with the mud telemetry. However, the telestrings are significantly expensive.

In order to address the aforementioned issues of both the low band width and the low SNR of the mud pulse telemetry and to avoid the expensive telestrings, it is here proposed to develop a novel ground structural health monitoring procedure based on the wave propagation characteristics of the DSS to monitor the downhole drill string vibrations in a real time. In this procedure, torsional and longitudinal waves generated from the BHA excitations and propagating through the drill strings are measured with an array of transducers installed at the top drill string on the ground.

In order to develop the proposed ground structural health monitoring procedure, a modeling approach to analyze the wave propagation through the DSS from the BHA to the ground needs to be investigated first. When an analytical approach is used to model the wave propagation, it is difficult to include detailed modeling information such as the effects of mud or the geometries of complex pipe joints, while a full 3-D numerical approach is significantly expensive due to its large model size to analyze the long DSS.

In order to address the aforementioned challenge, the Acoustic Transfer Functions (ATFs) identified by using the Hybrid Analytical/Two-Dimensional Finite Element Method (2D HAFEM) have been developed to analyze the wave propagation of the DSS. Then, it is here proposed that the HAFEM-based ATFs are used to predict vibration excitation force and torque at the BHA from the acoustic measurements made on the ground.

The HAFEM modeling approach uses an analytical wave solution in the axial direction of the pipe, while its response in the cross-sectional directions is represented by using a FE approximation. Thus, it is more computationally efficient than conventional full 3-D numerical modeling approaches, while including the detailed modeling information.

The analytical, one-dimensional (1-D) ATF modeling approach was experimentally validated with a drill pipe system of two hollow pipe sections connected in the middle. Then, the 2-D HAFEM was proposed for the modeling of a DSS when filled with a fluid (e.g., water) and it was validated by comparing the 2-D HAFEM results of a single, water-filled pipe with experimental data. In this article, the analytical 1-D ATF and 2-D HAFEM approaches are briefly summarized in the following section. Then, this article focuses on the estimation of the input forces and torques at the BHA of a DSS using the HAFEM-based ATF models of the DSS from acoustic data measured at the other end of the DSS with arbitrary boundary conditions.

For the validation of the proposed structural monitoring method, full 3-D FE analyses and experiments with a DSS of two pipes connected in the middle have been conducted. It is shown that the force and torque estimated by using the HAFEM-based ATF approach match well with the excitation force and torque except resonance frequencies.

2 THEORY

2.1 Acoustic Transfer Functions

For longitudinal wave propagation in the single pipe section in Fig. 1(a), the relation between the two sets of the acoustic variables at both the ends of the pipe can be written as

\[
\begin{bmatrix}
  u_{z1} \\
  N_{z1}
\end{bmatrix}
= \Psi_{L1} \begin{bmatrix}
  u_{z0} \\
  N_{z0}
\end{bmatrix}
= \frac{1}{2} \begin{bmatrix}
  e^{i k_{l1} d_l} + e^{-i k_{l1} d_l} \\
  \frac{1}{i E_1 S_l k_{l1}} (e^{i k_{l1} d_l} - e^{-i k_{l1} d_l}) - i E_1 S_l k_{l1} (e^{i k_{l1} d_l} - e^{-i k_{l1} d_l}) \\
  -i E_1 S_l k_{L1} (e^{i k_{l1} d_l} - e^{-i k_{l1} d_l}) \\
  (e^{i k_{l1} d_l} + e^{-i k_{l1} d_l})
\end{bmatrix} \begin{bmatrix}
  u_{z0} \\
  N_{z0}
\end{bmatrix},
\]

(1)
where \( u_z \) is the longitudinal displacement in the \( z \)-direction, \( \omega \) is the angular frequency, \( k_L \) is the longitudinal wave number defined as \( k_L = \omega / c_L \), \( c_L \) is the longitudinal wave speed defined as \( c_L = (E/\rho)^{1/2} \), \( E \) is the Young’s modulus, \( \rho \) is the density, and \( S \) is the cross-sectional area.

**Fig. 1** – Acoustic variables for modeling of single drill pipe section: (a) Longitudinal and torsional waves, and (b) flexural waves.

Similar to Eqn. (1), the torsional wave variables at both the ends of the pipe in Fig. 1(a) can be represented as:

\[
[\beta_{\theta L}] = \Psi_{T,1} [\beta_{\theta 0}] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ ik_{T} & -ik_{T} & k_{T} & -k_{T} \\ -E_{T}I_{r}k_{F}^{2} & -E_{T}I_{r}k_{F}^{2} & E_{T}I_{r}k_{F}^{2} & E_{T}I_{r}k_{F}^{2} \\ iE_{T}I_{r}k_{F}^{2} & -iE_{T}I_{r}k_{F}^{2} & -E_{T}I_{r}k_{F}^{2} & E_{T}I_{r}k_{F}^{2} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ E_{T}I_{r}k_{F}^{2} \\ iE_{T}I_{r}k_{F}^{2} \end{bmatrix}
\]

where \( \beta_{\theta} \) is the torsional angular displacement, \( c_T \) is the torsional wave speed defined as \( c_T = (G/\rho)^{1/2} \), \( G \) is the shear modulus, \( J \) is the torsional rigidity of the pipe, and \( k_T \) is the torsional wave number defined as \( k_T = \omega / c_T \).

The flexural wave relation between the two sets of the variables at both the ends of the pipe in Fig. 1(b) can be represented as:

\[
[\begin{bmatrix} u_{r1} \\ \beta_{z1} \\ M_{zz1} \\ Q_{r1} \end{bmatrix}] = \Psi_{FC,1} \Psi_{FZ,1} \Psi_{FC,1}^{-1} [\begin{bmatrix} u_{r0} \\ \beta_{z0} \\ M_{zz0} \\ Q_{r0} \end{bmatrix}]
\]

where

\[
\Psi_{FC,1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ ik_{F1} & -ik_{F1} & k_{F1} & -k_{F1} \\ -E_{T}I_{r}k_{F1}^{2} & -E_{T}I_{r}k_{F1}^{2} & E_{T}I_{r}k_{F1}^{2} & E_{T}I_{r}k_{F1}^{2} \\ iE_{T}I_{r}k_{F1}^{2} & -iE_{T}I_{r}k_{F1}^{2} & -E_{T}I_{r}k_{F1}^{2} & E_{T}I_{r}k_{F1}^{2} \end{bmatrix}
\]

and

\[
\Psi_{FZ,1} = \begin{bmatrix} e^{ik_{F1}d_1} & 0 & 0 & 0 \\ 0 & e^{-ik_{F1}d_1} & 0 & 0 \\ 0 & 0 & e^{ik_{F1}d_1} & 0 \\ 0 & 0 & 0 & e^{-ik_{F1}d_1} \end{bmatrix}
\]
In Eqn. (3), \(u_r\) is the transverse (or flexural) displacement, \(k_F\) is the flexural wave number defined as \(k_F = (\rho S \omega EI_F)^{1/4}\), and \(I_F\) is the second moment of the cross-sectional area. In addition, the flexural angular displacement, the flexural moment, and the shear force are represented as

\[
\beta_z = \frac{\partial u_r}{\partial z},
\]

\[
M_{zz} = EI_F \frac{\partial^2 u_r}{\partial z^2},
\]

\[
Q_{zr} = -EI_F \frac{\partial^3 u_r}{\partial z^3}.
\]

By combining Eqns. (1), (2), and (3), the 8 by 8 AFT matrix for the longitudinal, torsional, and flexural waves propagating through the pipe in Fig. 1 is obtained as

\[
U_{z=d_i} = \begin{bmatrix} \Psi_{L,1} & 0 & 0 \\ 0 & \Psi_{T,1} & 0 \\ 0 & 0 & \Psi_{F,1} \end{bmatrix} U_{z=0} = \Psi_1 U_{z=0} \tag{4}
\]

where

\[
U = \begin{bmatrix} u_z & N_{zz} & \beta_\theta & T & u_r & \beta_z & M_{zz} & Q_{zr} \end{bmatrix}^T.
\]

Fig. 2 – Longitudinal, torsional, flexural wave propagation in composite pipe system with multiple sections.

For a composite pipe system with multiple pipe sections as in Fig. 2, the matrix equation in Eqn. (4) can be applied to each pipe section. Then, the relation of the translational displacements, angular displacements, moments, and shear forces between \(z = 0\) and \(z = d\) for the entire pipe system can be obtained by multiplying all the matrices as

\[
U_{z=d} = \Psi U_{z=0} \cdot \Psi = \Psi_1 \Psi_{N-1} \Psi \cdots \Psi_1 \tag{5}
\]

2.2 Hybrid Analytical/Two-Dimensional Finite Element Method

Fig. 3 illustrates a 2-D HAFEM model of a pipe. In the cross-sectional directions, a finite element (FE) approximation is used, while an analytical solution is applied to the axial direction.
Then, the 2-D HAFEM equation of motion (EOM) for the pipe in Fig. 3 can be obtained as

\[
\frac{\partial^2 u}{\partial z^2} + K_{zz} \frac{\partial^2 u}{\partial z^2} + K_{(x+y)z} \frac{\partial^2 u}{\partial z^2} + M \frac{\partial^2 u}{\partial t^2} = f,
\]

where \( K \) is the stiffness matrix, \( M \) is the mass matrix, \( u \) is the nodal displacement vector, and \( f \) represents the external force vector.

In order to identify the wave propagation characteristics of the pipe, wave numbers are determined by substituting an assumed wave solution into Eqn. (6) as a function of frequency with no external excitation force. For this free vibration condition, an eigenvalue problem can then be derived as

\[
\left(-\frac{K_{zz}}{\omega^2} + ikK_{(x+y)z} + K_{x+y} - \omega^2 M\right)u_0 = 0.
\]

For a non-trivial solution, the determinant inside the brackets in Eqn. (7) should be zero to obtain the characteristic equation. Then, the wave numbers can be identified by solving the characteristic equation as the function of frequency.

### 2.3 HAFEM-Based ATF Matrix

Although the HAFEM modeling approach is useful to understand the wave propagation characteristics of a fluid-filled, multi-layered pipe, the cross-sectional shape of the pipe should not change in the axial direction. In order to consider a composite pipe system assembled with multiple pipe sections with different cross-sections, it is proposed that an ATF matrix is derived from the HAFEM formation of each pipe section. In particular, the wave numbers for the longitudinal, torsional, and flexural waves (i.e., \( k_{L1+}, k_{L1-}, k_{T1+}, k_{T1-}, k_{F11+}, k_{F11-}, k_{F11+}, \) and \( k_{F11-} \)) in the analytical ATF matrix presented in Eqn. (4) are obtained from the 2-D HAFEM-based characteristic equation in Eqn. (7) of this pipe section. Then, the HAFEM-based ATF matrix of this pipe section can be obtained. The total matrix of the entire pipe system can then be obtained by multiplying all the matrices as

\[
U_{z=d} = \begin{bmatrix}
\Psi_{HL} & 0 & 0 \\
0 & \Psi_{HT} & 0 \\
0 & 0 & \Psi_{HFZ} \Psi_{HFC}^{-1}
\end{bmatrix} U_{z=0} = \Psi U_{z=0}
\]

where
\[
\Psi_{HL} = \frac{1}{(k_{L+} - k_{L-})} \left[ -k_L e^{ik_{L,d}} + k_L e^{ik_{L,d}} \frac{i}{ES} (e^{ik_{L,d}} - e^{ik_{L,d}}) ight],
\]

\[
\Psi_{HT} = \frac{1}{(k_{T+} - k_{T-})} \left[ -k_T e^{ik_{T,d}} + k_T e^{ik_{T,d}} \frac{i}{f} (e^{ik_{T,d}} - e^{ik_{T,d}}) \right],
\]

\[
\Psi_{HFC} = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
-iElk_{F1+}^2 & -iElk_{F1-}^2 & Elk_{FR+}^2 & Elk_{FR-}^2 \\
iElk_{F1+}^3 & iElk_{F1-}^3 & -Elk_{FR+}^3 & -Elk_{FR-}^3
\end{bmatrix},
\]

\[
\Psi_{HFZ} = \begin{bmatrix}
e^{ik_{F1,d}} & 0 & 0 & 0 \\
0 & e^{ik_{F1,d}} & 0 & 0 \\
0 & 0 & e^{ik_{FR,d}} & 0 \\
0 & 0 & 0 & e^{ik_{FR,d}}
\end{bmatrix}.
\]

\[2.4 \text{ Force and Torque Estimation in Pipe with Arbitrary Boundary Conditions}\]

From Eqn. (8), for the longitudinal wave, the HAFEM-based ATF matrix between the one end of a pipe and two sensor locations at \(z = x_1\) and \(z = x_2\) in the pipe are represented as

\[
\begin{bmatrix}
u_{x_1} \\
N_{x_1}
\end{bmatrix} = \begin{bmatrix}
\psi_{HL,x_1,11} & \psi_{HL,x_1,12} \\
\psi_{HL,x_1,21} & \psi_{HL,x_1,22}
\end{bmatrix} \begin{bmatrix}
u_{z_0} \\
N_{z_0}
\end{bmatrix},
\]

\[
\begin{bmatrix}
u_{x_2} \\
N_{z_2}
\end{bmatrix} = \begin{bmatrix}
\psi_{HL,x_2,11} & \psi_{HL,x_2,12} \\
\psi_{HL,x_2,21} & \psi_{HL,x_2,22}
\end{bmatrix} \begin{bmatrix}
u_{z_0} \\
N_{z_0}
\end{bmatrix}.
\]

The first rows of Eqns. (13) and (14) can be represented as

\[
\begin{bmatrix}
u_{x_1} \\
u_{x_2}
\end{bmatrix} = \begin{bmatrix}
\psi_{HL,x_1,11} & \psi_{HL,x_1,12} \\
\psi_{HL,x_2,11} & \psi_{HL,x_2,12}
\end{bmatrix} \begin{bmatrix}
u_{z_0} \\
N_{z_0}
\end{bmatrix}.
\]

Then, from Eqn. (15), the axial force, \(N_{z_0}\) at \(z = 0\) can be estimated in terms of the measured two axial displacements, \(u_{x1}\) and \(u_{x2}\) as

\[
N_{z_0} = \frac{-\psi_{HL,x_1,11}u_{x_1} + \psi_{HL,x_1,12}u_{x_2}}{\psi_{HL,x_1,11} \psi_{HL,x_1,12} - \psi_{HL,x_1,12} \psi_{HL,x_1,11}}.
\]

Note that Eqn. (16) is independent of the boundary conditions at the ends of the pipe.
The same procedure as in Eqn. (15) for the longitudinal wave case can be applied to obtain the matrix equation: i.e.,

$$
\begin{bmatrix}
\beta_{\theta x_1} \\
\beta_{\theta x_2}
\end{bmatrix} =
\begin{bmatrix}
\psi_{HT,x_1,11} & \psi_{HT,x_1,12} \\
\psi_{HT,x_2,11} & \psi_{HT,x_2,12}
\end{bmatrix}
\begin{bmatrix}
\beta_{\theta 0} \\
T_0
\end{bmatrix}.
$$

(17)

Then, the estimated torque at $z = 0$ can be represented as

$$
T_0 = \frac{-\psi_{HT,x_1,11}\beta_{\theta x_1} + \psi_{HT,x_1,12}\beta_{\theta x_2}}{\psi_{HT,x_1,11}\psi_{HT,x_1,12} - \psi_{HT,x_1,12}\psi_{HT,x_2,11}},
$$

(18)

where $\beta_{\theta 1}$ and $\beta_{\theta 2}$ are the measured torsional angular displacements at the two sensor locations. Similar to the longitudinal wave case, the estimated torque equation in Eqn. (18) is not dependent on the boundary conditions of the pipe.

![Fig. 4 - Experimental setup with two hollow pipes connected in middle.](image)

**Table 1 - Material properties and dimensions of two hollow pipes and one joint.**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus [GPa]</td>
<td>208</td>
</tr>
<tr>
<td>Density [kg/m$^3$]</td>
<td>7856</td>
</tr>
<tr>
<td>Structural damping coeff.</td>
<td>0.0044</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Outer diameter [mm]</td>
<td>77.03</td>
</tr>
<tr>
<td>Inner diameter [mm]</td>
<td>54.65</td>
</tr>
</tbody>
</table>

3 EXPERIMENTS AND FINITE ELEMENT ANALYSES

In order to validate the proposed method, numerical analysis results and experimental data were obtained with a pipe that was built by connecting two hollow pipes in the middle and hanged by steel cables at two locations as shown in Fig. 4. The material properties and dimensions of the pipes and the joint are presented in Table 1. A commercial FE software package, ANSYS was used to obtain the numerical analysis results of the pipe. A Brüel & Kjær (B&K) PULSE system (Model: 3560-B-130) was used for the acquisition of the experimental data along with the PCB tri-axial accelerometers (Model: 356A24) installed on the pipe by using super glue. In the
In the results below, the measured or numerically-predicted Frequency Response Function (FRF) of the acceleration signal (at \( z = x \)) to the input excitation force or torque (at \( z = 0 \)) is used to estimate the force or torque. Thus, the estimated force or torque value is 1 in the results below. In “real” drill string systems, direct acceleration signals (without the normalization with the input excitation) will be used to predict the excitation force and torque accurately.

**Fig. 5** - Axial (or longitudinal) force estimated by applying HAFEM-based ATF approach to experimental and ANSYS FRF data of drill pipe in Fig. 4 for “longitudinal” excitation case: (a) Estimated axial force and (b) Axial force difference.

**Fig. 6** - Torque estimated by applying HAFEM-based ATF approach to ANSYS FRF data of drill pipe in Fig. 4 for “torsional” excitation case: (a) Estimated torque and (b) Torque difference.
Fig. 7 - Effects of sensor spacing obtained by applying HAFEM-based ATF approach to ANSYS FRF data of drill pipe in Fig. 4: (a) Longitudinal excitation case and (b) Torsional excitation case. Note that the y-axis is ranged from 0 to 3 dB, which is different from the previous plots.

The axial (or longitudinal) force and torque estimated by using the HAFEM-based ATF approach are well matched with the input axial force and torque (1 N or 120 dB, 1 Nm or 120 dB) mostly within the 3 dB difference as shown in Figs. 5 and 6. The difference of the axial force estimated by using the experimental data is relatively high in low frequencies, in particular, below 100 Hz in Fig. 5(b) since the experimental data include the effects of the hanging cables that are not modeled in both the HAFEM-based ATF approach and the ANSYS model. Additionally, the difference of the longitudinal force and torque is high at the resonance frequencies (e.g., 278 Hz and 470 Hz for the longitudinal excitation case in Fig. 5, and 174 Hz and 276 Hz for the torsional excitation case in Fig. 6). At the resonance frequencies, the strong standing wave patterns are generated, which results in the small difference of the two terms in the denominator of Eqn. (16) or (18), while each of these terms is large. The small difference of these two large terms causes the excitation estimation error. Further investigation will be made to minimize the estimation error in the near future.

Fig. 7 shows the effects of the sensor spacing between two sensors. As shown in Fig. 7, the axial force and torque differences decrease in most of frequencies as the sensor spacing is getting larger for both the longitudinal and torsional excitation cases.

5 CONCLUSIONS

In this article, the HAFEM-based ATF approach is proposed to estimate the excitation force and torque at the BHA of a DSS from an acoustic measurement on the ground. The HAFEM-based ATF approach results in the computationally-efficient models by combining the finite element approximation and the analytical wave solution approach. In addition, the HAFEM-based ATF approach can be applied to model the fluid-filled or fluid-surrounded, multi-layered composite pipes with arbitrary boundary conditions. In this article, the experimental and numerical data are used to validate the excitation estimation procedure. For the longitudinal excitation case, the estimated force is well matched with the excitation force except at resonance frequencies. The difference between the estimated and excitation forces and torques is also high at low frequencies, e.g., below 100 Hz, due to the cables to hang the pipe system. As for the torsional excitation case, the predicted torque is also matched well with the excitation torque within 3 dB except the resonance frequencies. In the near future, the effects of structural resonances and mechanical contacts between the drill pipe and the walls of a drilled well on the estimated force and torque
will be investigated further to minimize the estimation error. Additionally, the proposed monitoring method will be validated through a field test with a real DSS at the Riverside Campus, College Station, TX, USA.

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7 REFERENCES
